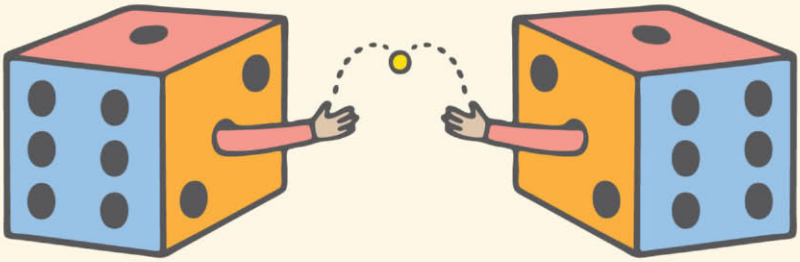

Nicolas Gisin

Quantum Chance

Nonlocality, Teleportation and Other Quantum Marvels

Foreword by Alain Aspect

Springer

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Foreword

"Love at first sight!" was how Nicolas Gisin described his emotion when he learned about Bell's theorem. When I heard this, I relived an autumn day of 1974 when I was immersed in study of John Bell's paper, little known at the time, and understood that it was possible to render an experimental verdict on the debate between Bohr and Einstein on the interpretation of quantum mechanics. Even though a few physicists knew of the problem raised by Einstein, Podolsky, and Rosen (EPR), not many had heard of Bell's inequalities, and few were those who considered questions relating to the fundamental concepts of quantum mechanics worthy of serious attention. The EPR paper, published in 1935 in *Physical Review*, was readily available in university libraries, but the same could not be said for the paper by Bell, published in an obscure new journal that was destined to disappear after only four issues. In those pre-internet days, papers not published in the major journals had to rely on photocopies for their dissemination. I had got my own copy from a file put together by Christian Imbert, a young professor at the *Institut d'Optique*, on the occasion of a visit by Abner Shimony, invited to Orsay by Bernard d'Espagnat. But once under the spell of Bell's ideas, I decided that my doctoral thesis would deal with experimental tests of Bell's inequalities, and Imbert accepted to take me under his wing.

In Bell's (impressively clear) paper, I was able to identify the crucial challenge for experimentalists: altering the orientations of the polarization detectors while entangled particles were still propagating from their source into the measurement regions. The point was to preclude influence of the polarizer orientations either on the emission mechanism or on the measurement, by application of the principle of relativistic causality, which forbids physical effects from propagating faster than the speed of light. Such an experiment would be able to scrutinise the essence of the conflict between quantum mechanics on the one hand and the world view held by Einstein on the other. Einstein defended local realism, which combines two principles. First, that there exists a *physical reality* of a system. Second, that a system cannot be influenced (the *locality assumption*) by anything that happens to another system separated from the first by a spacelike interval of spacetime, since those two systems would have

to communicate with influences propagating faster than light. Eventually, our experiments confirmed the predictions of quantum mechanics, forcing physicists to give up local realism, the view of the world defended so convincingly by Einstein. But should we give up realism or give up locality?

The idea that one should give up the notion of physical reality is not one I find convincing, because it seems to me that the role of a physicist is precisely to describe the reality of the world, and not just to be able to predict the results that show up on our measurement devices. But then, if quantum mechanics is confirmed on this count—as indeed seems unavoidable today—does that mean we must accept the existence of nonlocal interactions, in apparent violation of Einstein's principle of relativistic causality? And is there any hope of exploiting this quantum nonlocality to transmit a usable signal, e.g., to switch on a lamp or place an order at the stock exchange, that would travel faster than light? But this is where another characteristic feature of quantum mechanics comes into play, namely the existence of *fundamental quantum indeterminism*. This amounts to the absolute impossibility of influencing the actual result of any specific experiment whenever quantum mechanics predicts that several results are possible. It is true that quantum mechanics can be used to make very accurate calculations of the probabilities of the various possible results, but these probabilities have only a statistical meaning when the same experiment is repeated many times, and they tell us nothing about the result of any specific experiment. It is this *fundamental quantum randomness* which forbids the possibility of faster-than-light communication.

Among the many popular accounts of recent progress in quantum physics, the present book by Nicolas Gisin takes a definite line here, stressing the key role played by this fundamental quantum randomness, without which we might dream one day of designing a superluminal telegraph system. If it ever became a reality, this mythical invention of science fiction would require a radical revision of physics as we know it today. Naturally, the aim here is not to suggest that there might be untouchable and immutable physical laws, beyond any form of revision—quite the contrary, I am personally convinced that every physical theory will one day be superseded by one of wider scope. But some of our theories are so fundamental that their revision would involve a truly far-reaching conceptual revolution. And although we all know of a few examples of such revolutions through human history, they are nevertheless so exceptional that they should not be envisaged lightly. In this context, the explanation as to why quantum nonlocality, no matter how extraordinary it may be, cannot overthrow the principle of relativistic causality which forbids superluminal communication seems to me a particularly important feature of Nicolas Gisin's book.

The fact that this book adopts a particular stance on this issue, in contrast to other popular accounts, should come as no surprise, since Nicolas Gisin has been one of the key players in the new quantum revolution that took place in the last quarter of the twentieth century. The first quantum revolution, at the beginning of the twentieth century, was based on the discovery of wave-particle duality. It provided a way of describing with great accuracy the statistical behaviour of atoms that make up matter, of the clouds of electrons that conduct the electric current in a metal or semiconductor, and of the billions and billions of photons in a beam of light. It has also provided tools to understand the mechanical properties of solids, whereas classical physics was unable to explain why matter, made up of positive and negative charges that attract one another, does not simply collapse. Quantum mechanics has given a precise quantitative description of the electrical and optical properties of materials, and offers the conceptual framework needed to describe phenomena as surprising as superconductivity and the strange properties of certain elementary particles. It was in the context of this first quantum revolution that physicists invented new devices like the transistor, the laser, and integrated circuits, which have brought us today into the age of the information society. But then towards the 1960s, physicists began to ask new questions that had been pushed aside during the first quantum revolution:

- How can we apply quantum physics with its purely statistical predictions to single microscopic objects?
- Do the astonishing properties of *entangled pairs of quantum objects*, as described in the 1935 EPR paper but never actually observed, really correspond to the way nature behaves, or have we reached the limits of quantum mechanics with this issue?

It was the answers to these questions, first given by experimentalists, then further refined by theoreticians, that launched the second and ongoing quantum revolution.[1]

The behaviour of individual quantum objects has been the subject of lively debate between physicists. For a long time, the majority of the physics community thought that the question itself made little sense, and that it was in any case of no importance since it seemed inconceivable that one could ever

[1] See, for example, A. Aspect: John Bell and the second quantum revolution, foreword of J. Bell: Speakable and Unspeakable in Quantum Mechanics: Collected Papers on Quantum Philosophy, Cambridge University Press (2004); J. Dowling and G. Milburn: Quantum technology: the second quantum revolution, Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences **361**, 1809, pp. 1655–1674 (2003).

observe a single quantum object, let alone control and manipulate it. In the words of Erwin Schrödinger:[2]

> It is fair to state that we are not experimenting with single particles, any more than we can raise Ichthyosauria in the zoo.

But since the 1970s, experimentalists have developed ways to observe, manipulate, and control single microscopic objects such as electrons, atoms, and ions. I still recall the enthusiasm at the international conference on atomic physics in Boston in 1980 when Peter Toschek presented the first image of a single trapped ion, observed directly by the fluorescence photons it emitted under laser illumination. Experimental progress has since led to direct observation of quantum jumps, thus ending decades of controversy. It has also demonstrated that the quantum formalism is perfectly capable of describing the behaviour of single quantum objects, provided that one interprets the probabilistic results of the calculations in the right way. As for the second question, concerning the properties of entanglement, the quantum predictions were first tested on pairs of photons, in a series of experiments that is gradually converging upon the ideal conditions dreamt of by theoreticians like John Bell. And these experiments have consistently validated the quantum predictions, however surprising they may seem.

Having put together an applied physics group working on optical fibres in the 1980s, and having always had a personal and theoretical interest in the foundations of quantum mechanics (although secretly, or at least discreetly with regard to his employer, since in those days raising questions of this kind was not necessarily considered a worthwhile occupation), it was quite natural that Nicolas Gisin should have been among the first to test quantum entanglement on photon pairs injected into optical fibres. With his detailed knowledge of optical fibre technology, he was able to use the commercial telecommunications network around Geneva to demonstrate that entanglement is maintained even at separations of several tens of kilometers, to the surprise of the experimenters themselves! He used several conceptually simple tests to bring out the absolutely astounding features of entanglement between remote events, and to implement a quantum teleportation' protocol. Combining his skills as a theoretician on quantum foundations and expert in optical fibre applications, he was among the first to develop applications of entanglement such as quantum cryptography or the production of truly random numbers.

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[2] E. Schrödinger: Are there quantum jumps? British Journal for the Philosophy of Sciences, Vol. III, p. 240.

This combination of talents is evident throughout this fascinating book, which succeeds in presenting the subtle issues of quantum physics in a language that remains accessible to the general non-scientific public, and without recourse to mathematical formalism. He explains entanglement, quantum nonlocality, and quantum randomness, and describes a number of their applications. But this is more than just a popular account, and quantum specialists will find deep discussions of these phenomena, the true nature and consequences of which still remain largely beyond our grasp, as the author points out. Concerning the question as to whether the experimental refutation of local realism compels us to abandon the notion of physical reality or the idea of locality,[3] I take the same line as Nicolas Gisin: even if the concept of local realism may have been consistent and intellectually satisfying, cutting it into two pieces and keeping only one of them is distinctly less so. How should one define the autonomous physical reality of a system that is localised in space-time if this system is affected by what happens to another system separated from it by a spacelike interval? This book suggests a less brutal solution by showing that, if one takes into account the existence of fundamental quantum randomness, a nonlocal physical reality can be allowed to coexist in a more peaceable way with the relativistic causality so dear to Einstein. In this manner, even those physicists who know of these issues will find matter for reflection in Nicolas Gisin's book. And as regards the non-specialist reader, discovering here the mysteries of entanglement and quantum nonlocality, she or he will be carried straight to the heart of the problem and learn of all its subtleties, explained with enlightening clarity by one of the world's leading experts.[4]

Palaiseau, May 2012 Alain Aspect

[3] We may forget the desperate solution that consists in rejecting the notion of free will, a step that would make human beings into mere puppets under the direction of goodness knows what kind of Laplacian determinism.

[4] In 2009, Nicolas Gisin was the first winner of the prestigious John Stewart Bell prize, attributed for research on foundational problems of quantum mechanics and their applications.

Preface

Had you lived at the time of the Newtonian revolution, would you have wished to understand what was going on? Today, quantum physics gives us the opportunity to live through a conceptual revolution of similar importance. This book aims to help you understand what is happening, without mathematics, but also without trying to conceal the conceptual difficulties. Indeed, while physics needs mathematics to explore the consequences of its hypotheses and to precisely calculate some of its predictions, mathematics is not needed to tell the great story of physics. For what is interesting in physics is not the mathematics but the concepts. So my purpose here is not to manipulate equations, but to *understand*.

Certain parts of the book will demand a genuine intellectual effort on the part of the reader. Everyone will understand something and no one will understand everything! In this field, the very notion of understanding is blurred. But I claim nevertheless that everyone can understand at least a part of the conceptual revolution that is under way, and take pleasure from that understanding. To achieve this, one must simply accept that not everything is going to be transparent, and certainly not start out with the opinion all too often voiced that understanding physics is a hopeless task.

If part of the discussion seems too difficult, just read on. What comes next may throw light on the matter. Or sometimes you will realise that it was just some subtle point slipped in for my physicist colleagues, for they too may take pleasure in reading this book. And if necessary, go back later and reread those passages that caused you problems. The important thing is not to understand everything, but to acquire an overview. At the end of the day, you will find that one really can understand quite a lot of quantum physics without the need for mathematics!

Quantum physics has often been the subject of verbose interpretations and fuzzy philosophical dissertations. In order to avoid such pitfalls, we shall only have recourse to common sense here. When physicists carry out an experiment, they are questioning an external reality. The physicist decides what question to ask and when to ask it. And when the answer comes back, for example, in the form of a little red light that comes on, they do not ask themselves whether

that light is really red or whether it is just an illusion of some kind. The answer is red' and that's the end of it.

The reader will see that certain anecdotes turn up in several chapters of the book. My experience as a teacher has taught me that it is often extremely useful to repeat certain important points in different contexts. Finally, the book makes no claims to historical accuracy. Any notes on my illustrious predecessors reflect only my own impressions, picked up over the 30 years of my life as a professional physicist.

Introduction

In our most tender age, we learn that, to interact with an object that lies beyond our reach, only two possibilities are open to us. Either we move ourselves over to it, crawling toward it as babies do, or else we procure some long object like a stick which allows us to extend our reach. Later on we discover that more sophisticated mechanisms can also be put into effect, like dropping a letter into the mail box. The letter will be collected by a postman, sorted by hand or by machine, carried by lorry, train, or plane, and finally delivered to the door of the person whose name features on the envelope. Internet, television, and many other everyday examples teach us that, at the end of the day, any interaction and any communication between two spatially separated objects must propagate continuously from one point to the next by some mechanism which may be complex, but which always follows a continuous trajectory that can be identified in space and in time, at least in principle.

Nevertheless, quantum physics, which explores a world beyond the one we can perceive directly, asserts that objects spatially remote from one another can sometimes form a single unit. Indeed, for these systems, no matter how far apart their components, if we should prod one or the other of them, both will quiver! But how could we believe such a thing? Can such an assertion be put to the test? How should we understand it? And could we use this strange effect of quantum physics to communicate at a distance by exploiting these remote objects forming a single whole? These are the main questions we shall try to answer in this book.

I will attempt to share with you this fascinating discovery of a world that cannot be described by interactions propagating continuously from one point to the next, a world in which so-called nonlocal correlations become a fact of life. Along the way, we shall encounter the notions of irreducible chance, correlations, information, and even free will. We shall also see how physicists produce nonlocal correlations, how they exploit them to create absolutely secure keys in cryptography, and how these wonderful correlations can be used for quantum teleportation. Another aim of this book will be to illustrate the scientific method. How can one convince oneself that something totally counter-intuitive is actually true? What proof is required for such a change of

paradigm, and to accept a conceptual revolution of this kind? Stepping back for a moment, we shall see that the story of quantum nonlocality is actually rather simple and very human. And we shall also see that nature produces chance events (irreducibly chance-like!) which can occur at widely removed spatial locations without anything propagating from point to point along any path joining those locations. But we shall find that the chance-like character of these effects prevents any possibility of using this form of nonlocality to communicate, thereby saving from contradiction one of the fundamental principles of relativity theory according to which no communication can travel faster than the speed of light.

We are living in an extraordinary age. Physics has just discovered that one of our deepest intuitions, namely that objects cannot 'interact' at a distance, is not correct. The scare quotes on the word 'interact' remind us that we must clearly specify what we mean by that. Physicists explore the world of quantum physics, a world populated by atoms, photons, and other objects that seem quite mysterious to us. Missing out on this revolution without giving it further attention would be as much a shame as remaining ignorant of the Newtonian revolution or the Darwinian revolution, had we been their contemporaries. For the conceptual revolution taking place today is of no lesser importance. It completely overturns our previous pictures of nature and will doubtless give rise to a range of new technologies that will simply look like magic.

In Chap. 2, we present the notion of correlation which lies at the heart of the matter by discussing a game that we shall refer to as Bell's game. We show there that certain correlations cannot be produced if we are only allowed interactions propagating from point to point through space. This chapter will be crucial for the following, even though there will be no mention of quantum physics. It is very likely the most difficult chapter to understand, but the rest of the book will be there to help you.

We then ask how we should react if someone should ever win Bell's game, something that is apparently impossible even though it is an assertion of quantum physics, before confronting the idea of true chance in Chap. 3 and the impossibility of cloning quantum systems in Chap. 4. The following two chapters introduce this strange theory of quantum physics, first considering the theoretical concept of entanglement, then describing the relevant experiments and drawing the inevitable conclusion that *nature is nonlocal*.

But before accepting this conclusion, we shall ask whether it really is unavoidable. In Chap. 9, we shall survey many imaginative attempts by physicists to save a local description of nature. This story is still hot news and very much a topic of current affairs in the world of physics. Furthermore, it illustrates the natural cunning of the physicist! We continue our story in Chap. 10 by describing some of the fascinating research that is still ongoing. This will get us right up to date in the world of scientific research.

What Use is it?

This is the question I am most often asked. It is almost as though one should never do anything that has no immediate application. I could answer: "What's the point of going to the cinema?" True, I am paid to do this research I love so dearly, while I have to pay to go to the cinema, so I try to come up with a more politically correct reply. But quite frankly, the best answer is simply that it is so fascinating! Although I direct an applied physics group, I don't leap out of bed in the morning hoping to invent some new gadget. I am simply fascinated by physics! Just to understand nature, and in particular to understand how it can produce nonlocal correlations, is sufficient motivation. But then, why do I work in an applied physics group? Is it simple opportunism? In fact, there is a very good reason for being concerned with applications even when, and perhaps especially when, our deepest motivation is to understand the concepts. For a new concept of this relevance is sure to have consequences. For one thing, it will necessarily open up new prospects on the practical level. And the more revolutionary the concept, the more futuristic the applications. The great advantage of working on potential applications is precisely that it provides a tool for testing the underlying concepts. In addition, once an application has been identified, no one can deny the relevance of the concept! For how could one deny the relevance of a concept that underpins a working application in the real world?

The story of quantum nonlocality provides a first-rate illustration of this. Up until the first application, entanglement and nonlocality were largely ignored, even maligned as purely philosophical issues by the vast majority of physicists. Courage and even a certain audacity[5] was required by anyone who wished to consider these things before 1991. Almost no academic positions were attributed to this line of investigation, while today absolutely everyone is following the results of this research. Naturally, the governments that finance these research centers are more concerned with the quantum technologies than the concepts that underlie them, but the important thing is that students in these centers should be learning about this new physics.

Chapter 7 presents two applications that have already been commercialised: quantum cryptography and quantum random number generators. Finally, quantum teleportation, the most surprising of all the applications, will be described in Chap. 8.

[5] When Alain Aspect was beginning his research career, he went to see John Bell and suggested carrying out Bell's experiment. To this Bell replied: "Do you have a permanent post?" With all his experience, Bell knew that it was risky for any young physicist to work on a subject that was held in such contempt by the scientific establishment.

Acknowledgement

Having written this book, I think of all my students and other collaborators who have provided such stimulating interactions. I would also like to thank all those who read and criticised the first versions, and in particular the editor of the French edition, Nicolas Witkowski, and also to Stephen Lyle for the English translation. This book owes much to their patience and skills. Thanks also to the *Fonds national suisse de la recherche scientifique* and to Europe for generous funding of my research lab, and to the University of Geneva for being such a pleasant place to work. Finally, I thank providence for allowing me to live in such an exciting age for physics, and to contribute to it in my modest way.

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1
Appetiser

Before presenting the main theme of this book, I would like to begin with two short stories that will help to set the scene. One is a true story about something that happened in the past, while the other is pure fiction, but might actually come about in the near future.

Newton: So Great an Absurdity

Everyone has heard of Newton's universal theory of gravity, according to which all objects attract one another in a way that depends on their masses and the distance between them (or more precisely, the inverse square of that distance, but that will not be important here). For example, the Sun and the Earth are bound together by an attractive force that balances the centrifugal force and holds the Earth on an approximately circular orbit around the Sun. The same goes for the other planets, for the Earth–Moon system, and even for the whole of our galaxy, which revolves around the center of a cluster of galaxies.

But let us focus on the Earth–Moon system. How does the Moon know that it must be attracted by the Earth in a way that depends on the mass of the Earth and the separation between the two? For that matter, how does the Moon know the mass of the Earth and its distance away from us? Does it use some kind of measuring stick, like the baby mentioned earlier? Or does it throw little balls of some kind? Indeed, does it communicate in some special way? This apparently childish question is actually deadly serious. In fact, it greatly intrigued Newton, for whom the hypothesis of universal gravitation, although he discovered it himself and it had brought him such fame, was so absurd that no healthy mind could take it seriously (see box below).

But for the moment, suffice it to say that Newton's intuition was correct, even though it took several centuries and the genius of Einstein to fill the conceptual gap and provide a satisfactory answer. Today, physicists know that the action at a distance which occurs in gravitation, or indeed in the interaction between two electric charges, is not altogether instantaneous. Rather, it results from the exchange of messengers, whence the 'little balls' conjecture mentioned

N. Gisin, *Quantum Chance,*
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above turns out to be the right one. These messengers are tiny particles to which physicists attribute names. The messengers for gravity are called gravitons, while those for electric forces are called photons.

Box 1. Newton. That Gravity should be innate, inherent and essential to Matter, so that one Body may act upon another at a Distance through a Vacuum without the mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity, that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it.[1]

In this way, since Einstein, physics has described nature as a set of localised entities which can only interact with one another contiguously from point to point through space. This idea certainly fits with our intuition of the world, as it would with Newton's. But today, physics is also based on a second theoretical foundation, namely quantum physics, which describes the world of atoms and photons. Einstein was also involved in this discovery. In 1905, he interpreted the photoelectric effect as being due to bombardment by particles of light, or photons, which eject electrons from the surface of a metal by interacting mechanically with them, by direct contact, like billiard balls. But as soon as the full blown quantum theory was developed and formulated, Einstein quickly adopted a more critical stance, for he realised that this strange new theory would reintroduce a new form of action at a distance.[2] Like Newton three centuries before him, Einstein rejected this hypothesis which he took to be absurd, and which he described as a spooky action at a distance.

Today, quantum mechanics is well established at the very heart of modern physics. And it does indeed contain a form of nonlocality that would probably not have pleased Einstein, even though it is very different from the nonlocality that so bothered Newton. Moreover, this form of quantum nonlocality is well supported by experiment. There are even promising applications for it in cryptography and it makes possible the quite astonishing phenomenon of quantum teleportation.

A Peculiar Nonlocal Telephone

Here is a little piece of science fiction, although not nearly so futuristic as it may seem. In fact, technology will soon make it a reality. Imagine a 'telephone' connection between two speakers to whom we shall refer, according to the tradition, by the names of Alice and Bob, which correspond to the first

[1] Cohen, B., Schofield, R.E. (Eds): *Isaac Newton Papers and Letters on Natural Philosophy and Related Documents*, Harvard University Press (1958).

[2] Gilder, L.: *The Age of Entanglement. When Quantum Physics Was Reborn*, Alfred A. Knopf (2008).

two letters of the alphabet. As sometimes happens, the connection is rather poor, impaired by noise. In fact, the connection is so bad that Alice can hear nothing of what Bob is trying to tell her. All she can make out is a continuous noise—chzukscryprrskrzypczykrt. . . . Likewise, Bob hears only the same chzukscryprrskrzypczykrt. . . . In vain they shout into the receiver, fiddle with it, move around the room. But there is no improvement. How annoying! It is quite impossible to communicate with such a device, and it is abundantly clear that it does not deserve to be called a telephone.

But Alice and Bob are of course physicists. They each record 1 min of the noise produced by their device. In this way, Alice can prove to Bob that she has done her honest best, and conversely. But to their surprise, the noise recorded by the two friends turns out to be strictly identical. As the two recorders are digital, Alice and Bob can check that each bit of information on each recording is exactly the same. Incredible! The source of the noise must therefore be at the operator, or somewhere along the telephone line. As the noise is perfectly synchronised, they deduce that the source must be exactly midway along the line, in such a way that it arrives at the same time with both Alice and Bob.

They decide to test their hypothesis, namely that the cause of the noise is a defect, probably of electrical origin, exactly midway along the telephone line joining them. Alice thus extends her line by adding a long cable. The noise she receives should then be slightly delayed with respect to the noise received by Bob. But, no! Nothing changes. Not only is the noise still there and still identical at each end of the line, but it is still perfectly synchronised. Bob then cuts the phone line. But the noise just goes on!

How can we explain such a phenomenon? Is the wire on the receiver just a way to keep track of it in the flat? Is it just a mobile phone that happens to be attached to the wall out of pure convenience? Or is the noise produced by the receivers themselves rather than by some source situated between them? Could it be the explosion of some distant galaxy that produced the same noise in the two receivers? And how could one test these hypotheses? Bob, who knows something about electromagnetic waves, encloses himself within a Faraday cage, that is, a metal mesh structure that keeps out all radio waves. But the noise persists. Alice suggests that they move very far apart from one another. Then whatever mechanism is allowing the two receivers to communicate, the quality of the connection should decrease and finally disappear. But even then, moving apart has no effect on the amplitude of the noise.

Alice and Bob conclude that their receivers have recorded some very long sequence of noise which they then proceed to reproduce each time they are taken off the hook, delivering a sequence that is chosen very precisely as a function of the exact time. Then there should be no surprise to find that the two receivers always produce exactly the same noise.

Delighted with the success of their evidently scientific approach to the problem, Alice and Bob take the news of their discovery to their physics teacher, who duly commends them. But their teacher makes the following observation: "The assumption that the telephone receivers themselves make the same noise by virtue of some common cause, in fact the same noise previously recorded in the two receivers, is a hypothesis that can itself be tested. This is called a Bell test." Bell tests or Bell games will be presented in the next chapter. For the moment, let us just say that Alice and Bob rush back to their respective homes to carry out this Bell test on their receivers, and that the test fails. They repeat it several times, but always with the same result. The assumption of a common cause recorded in each of the two receivers is thus refuted.

Alice and Bob are left to wonder what mechanism could possibly allow their two receivers to produce this same noise, while widely separated, without communication, and without the noise being prerecorded in each receiver. But for all their efforts, they are unable to come up with any mechanism that could explain this phenomenon, so back they go to consult their teacher: "It's not surprising that you couldn't find a mechanism, because there simply isn't one. This is not a mechanical problem, but belongs to quantum physics. The noise is produced randomly, but it is a 'true' randomness. Each bit of the noise does not exist until the receivers produce it by a pure act of creation. Not only that, but this quantum randomness can manifest itself in several places at the same time, in your two receivers for example."

"But", exclaims Alice, "that's impossible. The signal must decrease with the separation between the two receivers, otherwise that would mean that one could communicate over arbitrary distances."

"Not only that", adds Bob, "but the perfect synchronisation would imply an arbitrarily high speed of communication, even faster than the speed of light, which is impossible."

But their teacher is unmoved: "You told me that the noise remains the same even when you shout into the receiver, even if you move around, spin round, or shake the device. So you see that the very fact that the same noise is produced randomly on both sides means you cannot use it to communicate. The other person learns absolutely nothing about what you are doing." And he concludes: "There is therefore no conflict with Einstein's relativity theory. You have shown once again that no communication goes faster than light."

Alice and Bob are speechless. Since their peculiar 'telephones' cannot be used to communicate, they are not really telephones, even though they might look like them. But how do they coordinate so as always to produce the same result without any communication, nor any prior agreement between them? And what about this suggestion of 'true' randomness which can manifest itself simultaneously in several different places? After a moment's silence, Bob finally

comes to his senses: "But if this is really what is happening, it should be possible to use this phenomenon. In which case, I will be able to build something, then play around with it until I understand how it works. After all, that's how I learnt how electricity works, how the trajectory of a ball will change when the ball is spinning, in fact, everything I've ever understood."

And their teacher agrees. This effect can be exploited to produce random numbers, and to secure confidential communications, which is known as quantum cryptography, and it can even be used for quantum teleportation. But first we must understand the central concept of this book, namely, nonlocality, which we shall do by discussing the idea of correlation and describing Bell's game.

2
Local and Nonlocal Correlations

The central concept in this book is *nonlocal correlation*. We shall see that this idea is closely related to the idea of true randomness, that is to the idea of events that are intrinsically unpredictable. Chance is already a fascinating subject in itself, but here we shall be talking about nonlocal chance. These are completely novel notions and very surprising, even revolutionary. And neither is it easy to grasp their relevance, which means that this chapter may well be the most difficult. But then the rest of the book is there to help you. In order to convince themselves that there really are nonlocal correlations and true randomness, physicists have invented a game, called Bell's game. Physicists are really big children who never stop taking their toys apart to understand what makes them tick.

But before introducing this game, we should begin by recalling what is meant by correlation. Science is essentially an exercise in observing correlations, then inventing explanations for them. John Bell used to say that correlations cry out for explanations.[1] We first present a simple example of correlations, then ask what kind of explanation can account for them. We shall see that there are in fact very few different types of explanation. If we limit ourselves to local explanations, that is, employing some mechanism that propagates continuously from point to point through space, there are actually only two different kinds.

Bell's game can then be used to study particular correlations. It is a game for two people who must work together in collaboration to obtain a maximum number of points. The rules of the game are exceedingly simple and it is easy to play, but the goal, a kind of nonlocal calculation, is not easy to apprehend at the outset. In fact, the point is not so much the game itself, as understanding how it works. And in this way we shall strike the heart of the matter—nonlocal correlations and the conceptual revolution currently under way.

But let us begin at the beginning, with the concept of correlation.

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[1] Bell, J.S.: *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press (1987), p. 152.

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Correlations

Every day we make choices which have consequences. Certain choices and their consequences are more important than others.

Certain consequences depend only on our choices, but many depend also on choices made by others. In this case, the consequences of our choices are not independent of each other: they are correlated. For example, the choice of menu for the evening meal depends among other things on the price of the produce at the local grocer's, and these prices are decided by others under various constraints. The menus of those inhabiting the same part of town will thus be correlated. If there is a special offer on fresh spinach, this vegetable is likely to appear more often on the menu. Another cause of correlation between menus is the influence of the choice made by the neighbour. If there is a long queue somewhere, we may be tempted to go and see what the attraction is, or else to avoid the queue. In both cases, there will be correlation, positive in the first case and negative in the second.

Let us push the example to the extreme. Imagine two neighbours, Alice and Bob once again. (We shall see that they play a similar role to the students in the story about the peculiar telephone.) Let us suppose that they always have the same evening meal, day after day. In other words, their evening menus are perfectly correlated. How could we explain such a correlation?

A first possibility is that Bob systematically copies Alice, and thus does not actually choose his menu, or conversely, that Alice copies Bob. Here then is a first possible type of explanation for the correlation: a first event influences a second event. This explanatory scheme can be put to the test, so let us behave like scientists and do just that. In thought at least, let us separate Alice and Bob so that they are really very far removed from one another, in two different towns on two different continents, but making sure that each has access to some local grocery store. To ensure that they cannot influence one another, we insist that Alice and Bob do their shopping at exactly the same moment. And better still, let us imagine that they are on two different galaxies. Under these conditions, it would be impossible for them to communicate, or even to influence one another unknowingly, like people who yawn.[2] But now imagine that the perfect correlation between their evening menus perdures. Now we cannot explain such a correlation by an influence, so we must find some other explanatory scheme.

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[2] We know that, in a group of people, if one yawns, that will trigger the same in others, whether they are aware of it or not. This is an example of an unconscious influence between people. However, the second person must necessarily see the first one yawning, so this kind of influence cannot propagate faster than light.

A second possible explanation is that the grocer's nearest to Alice and the one nearest to Bob offer one and the same product, whence there is in fact no choice whatever. Some time long ago, the two stores might have established a list of evening menus for the years to come. These menus might be different from one evening to the next, but every evening the two grocers respect the instructions featuring on their lists. This list might have been prepared by the manager of a chain of grocer's stores and communicated by email to all members of the galactic consortium. In this way, Alice and Bob necessarily end up with the same menu, day after day. According to this explanation, Alice and Bob's menus are determined by the same cause which occurred sufficiently long ago to influence both Alice and Bob, despite the huge distance that separates them. This common cause would have propagated continuously from point to point through space, without leaps or breaks. One speaks then of a common local cause, common because it arises from a shared past, and local because everything always happens locally and continuously, from one point in space to the next.

So here we have a possible logical explanation. Now think about it: is there any other possible explanation? Try your best to find a third kind of explanatory scheme that would account for the fact that Alice and Bob end up eating exactly the same meal every evening. That is, an explanation other than an influence of Alice on Bob or of Bob on Alice, and other than some common local cause. Is there really no other possible explanation? Surprising though that may seem, scientists have never found any third kind of explanation. All correlations observed in science, outside quantum physics that is, can be accounted for either by an influence of one event on another (explanation of the first type), or by common local causes, like the manager of the two grocery stores (explanation of the second type). In both types of explanation, the said influence or common cause propagates continuously from point to point through space, and in this precise sense, all such explanations are local. By extension, we speak of local correlations when we want to say that these correlations have some local explanation. In fact, we shall find that quantum physics provides us with a third possible explanation, and it is precisely the subject of this book. But outside quantum physics, there are only two types of explanation for all observed correlations, be they in geology or in medicine, in sociology or in biology. And these two types of explanation are local because they appeal to a chain of mechanisms that propagates continuously from one point to the next in space.

It is the quest for local explanations that has brought so much success to science. Indeed, science can be characterised by its incessant search for good explanations. And an explanation is considered good if it satisfies three criteria. The best known of these is accuracy. This is formalised in mathematical

equations which allow one to make predictions that can then be compared with observation and experience. However, it is my opinion that this criterion, albeit essential, is nevertheless not the most important. A second characteristic of a good explanation is that it tells a story. Every science lesson begins with a story. How else could one introduce new concepts, like energy, molecule, geological layer, or correlation? Until the advent of quantum physics, all these stories took place in a perfectly continuous manner in space and through time, and hence were local stories. The third criterion for a good explanation is that it cannot easily be modified. So a good explanation can be tested by experiments because it cannot easily be adapted to fit new experimental data that would otherwise contradict it. In Popper's words, it can be falsified.

But let us return to Alice and Bob and the perfect correlation between their evening meals. The wide separation between them rules out any attempt to explain by direct influence (type 1). How could we test an explanation by common local cause (type 2)? In our example, Alice has no choice in the matter. There is only one grocer's near where she lives and this store offers just one possible menu each evening. Such a situation, with no choice whatever, is too simple to be tested, so we must make our example a little more elaborate.

Imagine now that there are two grocery stores near Alice's home, one on the left when she goes out and one on the right. Likewise, there are two grocery stores near Bob's home, one on the left and one on the right. Alice and Bob still live on two different galaxies and cannot therefore influence one another. But let us imagine that, each time they both choose, quite by chance, to do their shopping in the store on the left, they always end up with the same menu. The only local explanation for this correlation is that the left-hand grocery stores share a list which evening after evening determines the sole available evening menu. For the grocery stores on the left, the situation is precisely the same as before. But the fact that there are several stores close to both Alice and Bob means that one can imagine a range of different correlations. For example, if Alice chooses the left-hand store and Bob the one on the right, we can once again imagine that they always end up with the same menu. Likewise if Alice goes to the right-hand store and Bob to the one on the left. We then conclude that the only local explanation for these three correlations, left–left, left–right, and right–left, is once again that these four stores share the same list of menus. But now imagine that, when Alice and Bob both go to the store on the right, they *never* have the same menu. Is that possible? Well, it does sound as though it would be difficult to arrange.

At this point we come very close to the spirit of Bell's game. So let us leave our grocery stores here. We must adopt a scientific approach and simplify the situation as far as possible. Instead of an evening menu, we shall speak of results, and since it suffices to consider just two possible results, we shall not require more than that.

Bell's Game

The manufacturer of this game supplies two apparently identical boxes as shown in Fig. 2.1. Each is equipped with a joystick and a screen. At rest, the joystick is in the vertical position. A second after the joystick is pushed to the left or the right, a result appears on the screen. The results are binary, that is, there are only two possible values, either 0 (zero) or 1 (one). Computer scientists say that the results are bits of information. For each box, the results seem to be random.

To play the game, Alice and Bob each take a box, synchronise their watches, then move some distance apart. At 9 a.m. exactly, and then every minute, they each push their joystick one way or the other, then carefully note the results displayed by their boxes along with the time and the direction they chose. It is important here that they each choose left or right every minute completely freely and independently. In particular, they are not allowed to keep making the same choice, nor to come to any prior agreement about the choices they will make. It is important that Alice should not know the choice made by Bob, and that Bob should not know the choice made by Alice. Note that they do

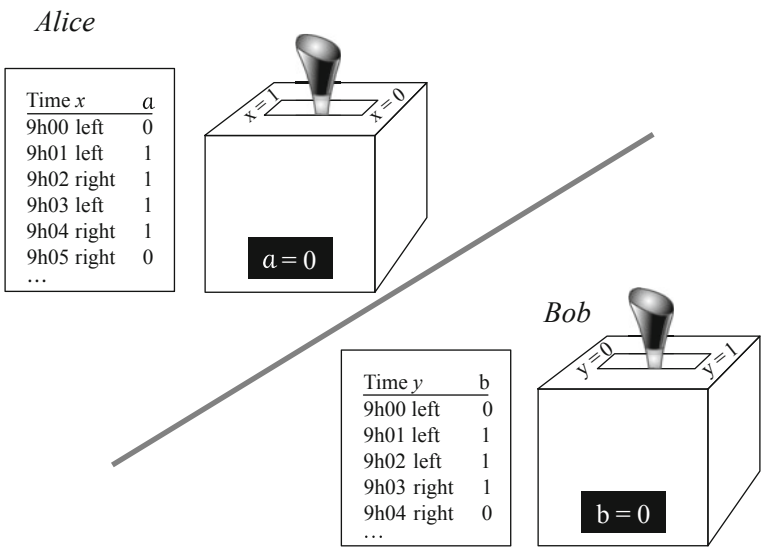

Fig. 2.1 Alice and Bob play Bell's game. Each has a box with a joystick. Every minute, they choose to push their joystick either toward the left or toward the right, whereupon each box displays a result. Alice and Bob carefully note the time, their choices, and the results produced by their boxes. At the end of the day, they compare their results and determine whether they have won or lost the game. Their objective is to understand how their boxes work, just as children learn by trying to understand their toys

not try to cheat, since their aim is to work out how the boxes supplied with the Bell game work.

They go on playing until 7 p.m., thus building up 600 pieces of data, with roughly 150 cases of left–left choices, and the same of left–right, right–left, and right–right. Then at the end of the day, they meet to calculate the points and their score according to the following rules:

1. Every time Alice pushes her joystick to the left or Bob pushes his to the left, or when both push their joysticks to the left, they get one point *if their results are the same*.
2. Every time Alice and Bob both choose to push their joysticks to the right, they get a point *if their results are different*.

The score is then calculated as follows:

- For each of the four combinations of choices left–left, left–right, right–left, and right–right, they first calculate the success rate, i.e., the number of points divided by the total number of trials, then add up these four success rates. The maximum score is thus 4, since there are four combinations of choices and each success rate can be at most equal to 1. For a score S, we shall say that Alice and Bob have won Bell's game S out of 4 times. Note that the score is an average and so can take any value between 0 and 4. For example, a score of 3.41 means that Alice and Bob have won on average 3.41 times out of 4, or 341 times out of 400.
- We shall see that it is easy to construct boxes allowing Alice and Bob to obtain a score of 3. So sometimes when we say that they win Bell's game, we shall mean that they win more often than 3 times out of 4.

To gain some familiarity with this peculiar game, let us think about the case where Alice and Bob do not in fact note down the results displayed by their boxes, but instead just write down whatever comes to mind. In short, they produce results by pure chance, quite independently of one another.[3] In this case, the success rates will all be 1/2. Indeed, half the time Alice and Bob write down the same result, and half the time opposite results, whatever their choices for the joystick. This means that the score for the Bell game will be $4 \times 1/2 = 2$. So to exceed a score of 2, Alice and Bob's boxes cannot be totally independent of one another, but must be coordinated in such a way as to produce correlated results.

[3] Note that the same reasoning applies also to the case where one of the two plays conscientiously while the other completely disregards the rules. In this case, the success rates will all be 1/2 once again and the total score 2.

Pursuing a little further, consider another example in which the two boxes always produce the same result 0, whatever the position of the joystick. In this case, Alice and Bob's choices have no influence on the result. It is easy to see that the success rates will be 1 for each of the three combinations left–left, left–right, and right–left, and 0 for the combination of choices right–right. The score will in this case be 3.

Before going on to analyse how the boxes work, let us introduce a modicum of abstraction. This will take us to the very heart of the notion of nonlocality.

Nonlocal Calculation: $a + b = x \times y$

Scientists love to encode the objects they are analysing using numbers, as we have done here with the results displayed by Bell's boxes. This helps them to focus on the essential without being confused by long sentences like "Alice pushed her joystick to the left and observed the result 0". It also allows them to carry out additions and multiplications, and we shall see that this can be used to encapsulate the notion of nonlocality in an extremely simple equation.

Let us focus first on Alice. Let x denote her choice and a the result. For example, $x = 0$ will mean that Alice chose to push her joystick to the left, and $x = 1$ will mean that she pushed it to the right. Likewise for Bob, let y denote his choice and b the result. With this notation, the little table below summarises the cases where, applying the rules of the Bell game, Alice and Bob get a point.

	$x = 0$	$x = 1$
$y = 0$	$a = b$	$a = b$
$y = 1$	$a = b$	$a \neq b$

So let us do a little elementary arithmetic, just for fun. It turns out that we can sum up the Bell game, with Alice and Bob each holding their box, widely separated from one another to avoid any possibility of copying, each making free choices and noting their results, in the form of a rather elegant equation:

$$a + b = x \times y,$$

that is, a plus b equals x times y. Indeed, the product $x \times y$ is always equal to 0 unless $x = y = 1$. The equation thus tells us that $a + b = 0$ unless $x = y = 1$.

Consider first the case where $x = y = 1$. In this case, $a + b = 1$, and since a and b are equal to 0 or 1, the equation $a + b = 1$ has only 2 solutions: either

$a = 0$ and $b = 1$, or $a = 1$ and $b = 0$. Hence, if $a + b = 1$, we certainly have $a \neq b$ and, according to the rules of the Bell game, Alice and Bob get a point.

Consider now the three other cases: $(x, y) = (0, 0)$, $(0, 1)$, or $(1, 0)$. We always have $x \times y = 0$ and the equation simplifies to $a + b = 0$. A first possible solution is $a = b = 0$. The second solution is $a = b = 1$. The second solution may look strange at first sight, because $1 + 1$ is normally equal to 2! But when we calculate with 0s and 1s (bits), the result must also be a 0 or a 1. Here, $2 = 0$ (mathematicians talk about calculating modulo 2). So the equation $a + b = 0$ is equivalent to $a = b$.

In conclusion, the beautiful equation $a + b = x \times y$ perfectly summarises the Bell game. Every time it is satisfied, Alice and Bob receive one point. And so you see that the quantum revolution can be exposed with very simple mathematics.[4]

This equation expresses the phenomenon of nonlocality. In order to win the Bell game systematically, the boxes must calculate the product $x \times y$. But if the choice x is only available to Alice's box and the choice of y is only available to Bob's box, this calculation cannot be done locally. At best, they may bet on $x \times y = 0$ and they will be right 3 times out of 4, whence they may obtain a score of 3. But any score greater than 3 requires a 'nonlocal' calculation of $x \times y$, because the two factors in the product exist only in places that are very far removed from one another.

Local Strategies for the Bell Game

Alice and Bob are in front of their respective boxes and each minute they make their choice freely and independently, carefully noting down their choices and the results displayed on the boxes. What could these boxes do to ensure that Alice and Bob get a good score?

Let us imagine that they are too far apart to influence one another. To achieve this, move Alice and Bob apart in our minds, so far from each other that no communication is possible. For example, separate them by a distance such that even light would take more than one second to go from one to the other, i.e., more than 300,000 km, roughly the distance from the Earth to the Moon. In this extreme case, it would be impossible for Alice, or rather for her box, to communicate her choice to Bob, or Bob's box. No account in terms

[4] As a good but somewhat unruly student, how many times during my studies did I ask my quantum physics teachers for explanations, only to hear that quantum physics cannot be understood because it requires such complex mathematics?

of communication or influence is thus possible and we must therefore find another explanation.

Let us begin by analysing the case where, by coincidence, the two joysticks are pushed toward the left. In this case, Alice and Bob only obtain a point if their results are the same. This is the same situation as for the customers at the grocery stores who always end up with the same menu if they choose the store on the left. We have already seen that, if we exclude all direct influence, this is only possible if the two stores leave absolutely no choice and simply impose the same menu. For the boxes in Bell's game, this means that, if the joysticks are pushed to the left, they both produce the same result. This result is predetermined at each minute, but can change from one minute to the next, just as the single available menus can vary from one evening to the next. Here we have an explanation for a maximal correlation in the case where the two joysticks are pushed toward the left. This is an explanation of the second type, that is, in terms of a common local cause. Indeed, the results predetermined each minute must be recorded in each box, that is, locally.

Let us pursue the analysis of this case a little further. The results originally recorded in the boxes may have been produced by a long series of coin tosses. From Alice's point of view, they thus look perfectly random. And the same goes for Bob. However, when they meet together and discover that they have always obtained the same result, they no will longer believe that this happened by chance. Unless perhaps this is nonlocal randomness? We shall come back to that.

Box 2. Chance. A result that occurs by chance is one that is unexpected. But unexpected for whom? Many things are unexpected, either because they result from processes that are too complex to be apprehended, or because we did not pay attention to all kinds of details that have influenced the result. However, a truly random result that occurs by 'true' chance is unexpected because it is *intrinsically* unpredictable. Such a result is not determined by one or other causal chains, however complex they may be. A truly random result is not predictable because, before it came into being, it just did not exist, it was not necessary, and its realisation is in fact an act of pure creation.

To illustrate this idea, imagine that Alice and Bob meet up by chance in the street. This might happen, for example, because Alice was going to the restaurant further down the same street and Bob to see a friend who lives in the next street. From the moment they decide to go on foot, by the shortest possible path, to the restaurant for Alice and to see his friend for Bob, their meeting was predictable. This is an example of two causal chains of events, the paths followed by Alice and Bob, which cross one another and thus produce what looks like a chance encounter to each of them. But that encounter was predictable for someone with a sufficiently global view. The apparently chance-like nature of the meeting was thus only due to ignorance: Bob did not know where Alice was going, and conversely. But what was the situation before Alice decided to go to the restaurant? If we agree that she enjoys the

benefits of free will, then before she made this decision, the meeting was truly unpredictable. True chance is like this.

True chance does not therefore have a cause in the same sense as in classical physics. A result subject to true chance is not predetermined in any way. But we need to qualify this assertion, because a truly chance-like event may have a cause. It is just that this cause does not determine the result, only the probabilities of a range of different possible results. It is only the propensity of a certain result to be realised that is actually predetermined.

According to the explanatory scheme for common local causes, every minute each box produces a predetermined result. For this kind of explanation, the list of results is pre-established and memorised by each box. We may think of each box as containing some kind of little computer with a large memory, a clock, and a program which reads off the next piece of data in the memory at one minute intervals.

Depending on the program, the result can either be independent of the position of the joystick or it can depend on it. But what programs are running in Alice and Bob's boxes? Are there not infinitely many, or at least a very large number of possible programs? In fact, there are not, because the simplification we have made in our scientific approach by sticking to binary choices and results limits the number of possible programs to 4 per box. Indeed, the program need only supply one result among 2 possible for each of the 2 possible choices. In Alice's box, these 4 programs are as follows:[5]

1. The result is always $a = 0$, whatever the choice of x.
2. The result is always $a = 1$, whatever the choice of x.
3. The result is identical to the choice, i.e., $a = x$.
4. The result always differs from the choice, i.e., $a = 1 - x$.

Likewise, there will be 4 possible programs for Bob's box. This means a total of $4 \times 4 = 16$ combinations of programs for both Alice and Bob. Naturally, the programs can change from one minute to the next, both in Alice's box and in Bob's, but at each minute, one of the 4 programs in Alice's box determines its result a and one of the 4 programs in Bob's box determines b.

Let us investigate these 16 possible combinations of programs and calculate the corresponding scores. Remember that the aim is to find the maximal possible score with a local explanation. We shall see that it is impossible to build boxes using local strategies to obtain a score greater than 3. At this point, you have the choice of simply taking my word for it and moving directly to

[5] Here the notion of program is to be taken in the abstract sense of saying *what results are produced by what data*. An abstract program can clearly be written in many ways, in various programming languages, and possibly with many unnecessary lines, in such a way that it may be difficult to see that two programs written differently do in fact correspond to the same abstract program.

Table 2.1 Scores for the 16 possible combinations of the 4 programs available for each box

Alice's program	Bob's program	Results for choice $(x, y) = (0, 0)$	Results for choice $(x, y) = (0, 1)$	Results for choice $(x, y) = (1, 0)$	Results for choice $(x, y) = (1, 1)$	Score
1	1	$a = 0, b = 0$	$a = 0, b = 0$	$a = 0, b = 0$	$a = 0, b = 0$	3
1	2	$a = 0, b = 1$	$a = 0, b = 1$	$a = 0, b = 1$	$a = 0, b = 1$	1
1	3	$a = 0, b = 0$	$a = 0, b = 1$	$a = 0, b = 0$	$a = 0, b = 1$	3
1	4	$a = 0, b = 1$	$a = 0, b = 0$	$a = 0, b = 1$	$a = 0, b = 0$	1
2	1	$a = 1, b = 0$	$a = 1, b = 0$	$a = 1, b = 0$	$a = 1, b = 0$	1
2	2	$a = 1, b = 1$	$a = 1, b = 1$	$a = 1, b = 1$	$a = 1, b = 1$	3
2	3	$a = 1, b = 0$	$a = 1, b = 1$	$a = 1, b = 0$	$a = 1, b = 1$	1
2	4	$a = 1, b = 1$	$a = 1, b = 0$	$a = 1, b = 1$	$a = 1, b = 0$	3
3	1	$a = 0, b = 0$	$a = 0, b = 0$	$a = 1, b = 0$	$a = 1, b = 0$	3
3	2	$a = 0, b = 1$	$a = 0, b = 1$	$a = 1, b = 1$	$a = 1, b = 1$	1
3	3	$a = 0, b = 0$	$a = 0, b = 1$	$a = 1, b = 0$	$a = 1, b = 1$	1
3	4	$a = 0, b = 1$	$a = 0, b = 0$	$a = 1, b = 1$	$a = 1, b = 0$	3
4	1	$a = 1, b = 0$	$a = 1, b = 0$	$a = 0, b = 0$	$a = 0, b = 0$	1
4	2	$a = 1, b = 1$	$a = 1, b = 1$	$a = 0, b = 1$	$a = 0, b = 1$	3
4	3	$a = 1, b = 0$	$a = 1, b = 1$	$a = 0, b = 0$	$a = 0, b = 1$	3
4	4	$a = 1, b = 1$	$a = 1, b = 0$	$a = 0, b = 1$	$a = 0, b = 0$	1

p. 19 (the section entitled *Winning at Bell's Game: Nonlocal Correlations*), or taking the time to convince yourself by working through the argument in the next paragraph, something I would strongly encourage you to do.

Let us begin with the combination of program 1 for Alice and program 1 for Bob. In this case the two results are always 0, i.e., $a = b = 0$, and Alice and Bob win Bell's game 3 times out of 4. Indeed, they only lose Bell's game when they both make choice 1. Now consider a second combination of programs, say program 1 for Alice, hence always $a = 0$, and program 3 for Bob, hence $b = y$. Consider in turn the 4 pairs of possible choices they can make. For $x = 0$ and $y = 0$, the results are (0,0) and so Alice and Bob win, in the sense of obtaining a point. For $x = 0$ and $y = 1$, the results are (0,1) and they lose, in the sense that they do not get a point. For $x = 1$ and $y = 0$, the results are (0,0) and they win. Finally, for $x = 1$ and $y = 1$, the results are (0,1) and they win once again, since for $x = y = 1$ the aim is to have different results. To sum up then, Alice and Bob once again score 3.

And now you may finish the argument yourself, considering one after the other the 14 remaining combinations of programs. Alternatively, you can consult the results presented in Table 2.1.

To sum up then, whatever the local strategy of the box manufacturer, and hence whatever the combination of programs, Alice and Bob can never win Bell's game more often than 3 times out of 4. Physicists like to express this result

in the form of an inequality. This is Bell's inequality.[6] Since this inequality is central to the theme of this book, I shall write it in full. Even if you do not fully grasp the meaning, it should be possible to appreciate the beauty, just as some of us appreciate the beauty of a musical score:

$$P(a = b|0,0) + P(a = b|0,1) + P(a = b|1,0) + P(a \neq b|1,1) \leq 3.$$

The symbol $P(a = b|x, y)$ reads as follows: the probability that a is equal to b in the case where the choices x and y have been made. Likewise, $P(a \neq b|1,1)$ reads as the probability that a differs from b in the case where the choice $x = y = 1$ has been made. Bell's inequality states what we have just found, namely that the sum of the 4 probabilities in the Bell game, which gives the score, is at most 3. So for local correlations, Bell's inequality is always satisfied.

Box 3. Bell's Inequality. Quite generally, the probability $P(a, b|x, y)$ may result from a statistical mixture of different possible situations. For example, a first possible situation, traditionally denoted by λ_1, can occur with a probability $\rho(\lambda_1)$, a second possibility λ_2 with probability $\rho(\lambda_2)$, and so on. These probabilities $\rho(\lambda)$ can also be used to analyse cases where we do not exactly know the real situation. In fact, we do not even need to know the probabilities of these situations. It is enough to know that different situations occur with different probabilities.

These situations λ can include the quantum state, usually denoted by ψ. In fact, they can include the whole past of Alice and Bob, or even the state of the whole universe, except for one thing: the choices x and y must be independent of λ. On the other hand, the λ can be much more limited, like the choices of Alice's and Bob's strategies in the Bell game. Historically, the λ were called local hidden variables, but it is better to consider them as the physical state of the systems, i.e., Alice and Bob's boxes, as described by any present or future theory. So Bell's inequalities tell us something about the structure of any future physical theory compatible with today's experiments. In short, the only assumption about the λ is that they contain no information about the choices x and y.

For each situation λ, the conditional probability can always be written as

$$P(a, b|x, y, \lambda) = P(a|x, y, \lambda)P(b|x, y, a, \lambda).$$

The locality assumption then states that, for any λ, what happens in Alice's box does not depend on what happens in Bob's box, expressed by $P(a|x, y, \lambda) = P(a|x, \lambda)$, and conversely, $P(b|x, y, a, \lambda) = P(b|y, \lambda)$. In conclusion, the assumption underlying all the Bell inequalities is found by averaging over all possible situations λ :

$$P(a, b|x, y) = \sum_{\lambda} \rho(\lambda)P(a|x, \lambda)P(b|y, \lambda),$$

where $\rho(\lambda)$ denotes the probability that the situation λ occurs.

[6] More precisely, it is the simplest in a family of Bell inequalities, equivalent to the CHSH inequality, named after its discoverers J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt: *Proposed experiment to test local hidden-variable theories*, Phys. Rev. Lett. **23**, 880 (1969). The other inequalities correspond to cases where there are more possible choices, more possible results, or more players.

Up to now we have assumed that Alice's and Bob's boxes each contain a program that determines the results as a consequence of the choices x and y. (Computer scientists would refer to x and y as input data.) But what would happen if these programs did not fully determine the results, but left some room for chance? For example, one could imagine that, from time to time, Alice's box chooses at random to apply program 1 or program 3. Or again, from time to time, the box might produce a random result. Can this help them to win Bell's game? Note first that producing a random result comes to the same as choosing at random between program 1 (which determines $a = 0$) and program 2 (which determines $a = 1$). It turns out that this serves no purpose. Bell's game involves a large number of repetitions and computation of an average. If at a given minute Alice's box chooses a program at random from a set of programs, the score will not be different from what is obtained if at each minute this box uses a specific program chosen at random from this set of programs. Assuming that each minute the boxes use a specific program is not therefore in any way restrictive. Including random strategies for Alice and Bob would not therefore help them to win Bell's game. Quite the opposite, in fact. As we have seen, if Alice and Bob's boxes independently produce results at random, they only obtain a score of 2.

In conclusion, no local strategy can be employed to win Bell's game more often than 3 times out of 4. Physicists would say that no local correlation can violate Bell's inequality. Put another way, if Alice and Bob nevertheless managed to win Bell's game more often than 3 times out of 4, there would be no local explanation for this phenomenon. As we have seen, there are only two types of local explanation, those based on influences propagating contiguously from point to point through space (type 1) and those based on common causes which also propagate from point to point through space from some shared past (type 2). But type 1 explanations are excluded by the large distance that separates Alice and Bob and, as we have just seen, type 2 explanations can never be used to win Bell's game more often than 3 times out of 4.

Winning at Bell's Game: Nonlocal Correlations

Now imagine that Alice and Bob play Bell's game for a long time and win on average much more often than 3 times out of 4. This is precisely what is made possible by the phenomenon of entanglement in quantum physics. But for the moment we shall leave this fascinating piece of physics to one side and simply consider the hypothesis that Alice and Bob win Bell's game very often. Now we have ruled out the possibility of their influencing one another, or of

their boxes communicating, even by means of some as yet unknown kind of wave (and we shall return to this important hypothesis later on). We have just seen that, if the boxes produce the result locally depending on the time and position of the joystick, and hence depending on the choice of the operator, then the game cannot be won more often than 3 times out of 4. In other words, winning more often than 3 times out of 4 is impossible if one only adopts local strategies, that is, if one only uses mechanisms that propagate from point to point through space.

This is why correlations allowing one to win Bell's game more often than 3 times out of 4 are said to be nonlocal. But then, how do Alice and Bob manage to do this with their boxes?

If one had asked a physicist before the advent of quantum physics, let us say prior to 1925, the answer would have been very simple: they would have said it was strictly impossible! To win Bell's game more often than 3 times out of 4, Alice and Bob, or at least their boxes, must be cheating in some way, either communicating with one another or influencing one another, possibly unconsciously, just as one can make someone yawn by yawning oneself. But without communication, our pre-quantum scientist would have said that it was simply impossible.

What about you? Can you see how to win Bell's game more often than 3 times out of 4? Do you believe it is possible? I am sorry to torment your grey matter with this game, but it really does lie at the very heart of the issue of nonlocality. We are perhaps in the same position as people in the Middle Ages when they were told that the Earth is round like a ball and that other people were living on the other side of it. But then how come they did not fall off? Today everyone knows that all objects, including people, fall toward the center of the Earth, not from top to bottom. People on the other side of the Earth are thus held to the ground in the same way as magnets are held on the fridge door. Thanks to the magnets, everyone understands that we are attracted to the Earth, so neither Australians nor Europeans will fall off.

But for Bell's game, what is the analogue of the magnet on the fridge door? What story can we tell in order to understand this? Unfortunately, I cannot give you an intuitive explanation of the way quantum entanglement allows us to win Bell's game more often than 3 times out of 4, but I would invite you to pursue our exploration of the world of atoms and photons, to play around with this peculiar game and try to imagine some entertaining and even useful consequences. Let us see what it means for our view of the Universe. Let us dismantle these correlations in the same way as a child might take its toys apart to find out what inner mechanisms they contain.

Box 4. John Bell: "I am a Quantum Engineer, but on Sundays I Have Principles". I was lucky enough to meet John Bell quite often. Here is the story of one of my first meetings with him.

"I am a quantum engineer, but on Sundays I have principles," began Bell at a rather unusual meeting in March 1983. Those were words I would never forget! John Bell, the famous John Bell, introduced himself as an engineer, one of those practical people who know how to make things work, whereas for me, recent proud recipient of a doctorate in theoretical physics, John Bell was a giant among theoreticians.

In 1983, the Vaud physical research society organised its annual training week, bringing together teachers and research physicists, consisting of a week in Montana, half ski and half lectures given by well known scientists. In that particular year, the subject was foundations of quantum physics and it was an opportunity to meet Alain Aspect, the first man[7] to have won Bell's game, and to spend some pleasant afternoons skiing with him. John Bell was invited, and indeed to have done otherwise would have been impossible, but he did not feature on the programme, i.e., on the lecture timetable, for motivations particular to this part of the physics community, and this was clearly an aberration. With another student, we asked Bell if he would give us an off-the-cuff presentation. At first he refused on the grounds that he did not have his slides with him, but in the end, Bell's clandestine talk did take place one evening after dinner, in a cellar hastily transformed into a lecture room with the spectators sitting on the floor. The engineer with principles explained how one can use physics in a pragmatic way to develop applications, to carry out difficult or entertaining experiments, and to identify empirical rules that work well in all practical situations, but that one should not lose sight of the overriding aim of science: to explain nature in a consistent way. Bell's message has haunted me ever since.

Winning Bell's Game Does Not Imply Communication

Let us suppose that Alice and Bob win Bell's game more often than 3 times out of 4, perhaps even every time. Could they use this to communicate?[8] Note that, since they are separated from one another by an arbitrarily great distance, such communication would imply the possibility of communicating at an arbitrarily high speed.

How could Alice communicate some piece of information to Bob? The only means available must go through the positioning of the joystick. For example, 'left' could mean 'yes' and 'right' could mean 'no'. But from Bob's point of

[7] The American physicist John Clauser had achieved a similar feat a few years earlier, but the boxes used did not rule out the possibility of information exchange. Moreover, they could only deliver one result, for example 0, the other result 1 being obtained by indirect measurements.

[8] One should not confuse the hypothesis of some kind of 'subtle' communication between the two boxes in order to win Bell's game with the possibility for Alice and Bob to use the correlation produced by their boxes to communicate with each other. The former would be a kind of hidden communication which we might refer to as an 'influence'. The second would allow Alice and Bob to communicate without needing to understand or control the internal operation of the boxes.

view, his box is just producing results randomly. Whatever the position of his joystick, the two possible results $b = 0$ and $b = 1$ occur equally frequently. And that remains true whatever the position of Alice's joystick. There is therefore no way to use the correlations in Bell's game to send a message from Alice to Bob, or conversely. It is only by comparing the results of the two boxes that one could notice the correlations. Recall the peculiar 'telephone' in Chap. 1.

It is thus impossible for Alice and Bob to use the two boxes to communicate.[9] It is only when Alice and Bob have been able to compare their results, hence only when they have stopped playing and met together at the end of the day, that they can ascertain whether or not they have won Bell's game. There is thus no connection between Alice and Bob that would allow them to communicate. Communicating solely through Bell's game would mean communicating without any physical entity carrying the message from emitter to receiver. It would be communication without transmission, i.e., an impossible communication, as explained in Box 5.

But could there be some kind of link, or 'invisible thread', that connects the two boxes, not to communicate, but simply allowing the two boxes to win Bell's game? If there were such a link, we would then understand the underlying 'trick'. That might be disappointing though, a simple magician's sleight of hand.

Box 5. No Communication Without Transmission. If one person, say Alice, wants to communicate a message to another, say Bob, she must first transcribe her message on some kind of physical medium. The message is then carried from one point to another by this physical medium, e.g., a letter, electrons, or photons. Bob then receives this physical medium and reads, or decodes, the message it carries. In this way, the message is transmitted contiguously from one point to another through space from Alice to Bob. Any other way of communicating a message would not be physical.

For instance, if Alice could choose a message and transcribe it on some physical medium, but nothing left her in the sense that no physical entity left the point of space where she happens to be, then there would be no way for her to communicate her message. Otherwise, as Newton had already understood (see Box 1 on p. 2), one would have communication without transmission. But it is impossible to communicate without something physical like matter, waves, or energy leaving Alice once she has chosen the message she wishes to communicate.

This is simply a matter of common sense. If one violated this principle by telepathy, for instance, one could communicate at an arbitrary speed. Indeed, since nothing would be needed to carry the message, the distance between Alice and Bob would be of no importance, and by arbitrarily extending this distance, one could have an arbitrarily high communication speed, for example, faster than light. But the impossibility of such a thing is even more fundamental than the theory of relativity itself, which forbids any speed greater than the speed of light: non-physical communication is impossible.

[9] Formally, a correlation $P(a, b|x, y)$ cannot be used to communicate if the marginal distributions do not depend on the input from the other part, i.e., if $\sum_b P(a, b|x, y) = P(a|x)$ and $\sum_a P(a, b|x, y) = p(b|y)$.

But for the physicist, it could spell the beginning of an important discovery. What constitutes this link? How does it work? How fast can it transmit the hypothetical hidden influences between the boxes? But for the time being, let us just note that no visible link is perceptible, and note also that our two boxes are so widely separated that no influence propagating at the speed of light would reach its destination in time. Moreover, Alice and Bob do not need to know where their partner is. They can pick up their box and take it to some unknown port of call.

Opening the Boxes

In 1964, when John Bell first presented his game in the form of an inequality, it was only a thought experiment. But since then, this game has been made a reality in many research laboratories. So let us open up these apparently magical boxes, since they allow us to win Bell's game.

When we look inside, we find a full complement of physical equipment, including lasers (red, green, and even some producing a beautiful yellow light), a cryostat (a kind of fridge for cooling something down to temperatures close to absolute zero, i.e., around −270°C), fibre optic interferometers (optical circuits for photons), two photon detectors (able to detect particles of light), and a clock (see Fig. 2.2). But all this is of little help to us.

Investigating this equipment more closely, we discover at the center of the cryostat, at the intersection of all the laser beams, a tiny crystal measuring a few millimeters across and looking like a little piece of glass. And this insignificant crystal certainly seems to be at the very heart of the setup. Indeed, when the joystick is pushed to the left or the right, it triggers a series of laser pulses that shine on the crystal and activate a piezoelectric element[10] in the interferometer connected to the crystal. This piezoelectric element moves a short distance in the same direction as the joystick. Then one of the two photon detectors is triggered, and the box subsequently generates a 0 or a 1. It is clear that the joystick causes something to happen in the crystal, by virtue of the laser pulses, then determines the state of the interferometer thanks to the piezoelectric element. Finally, the result is given by the detector. The two boxes are strictly

[10] When we put pressure on a piezoelectric element, it produces an electric potential difference, and conversely, when we apply a potential difference, the element is compressed. The two effects go hand in hand. One of the most familiar applications is the gas lighter. When pressure is exerted, it produces a small voltage which discharges suddenly in the form of sparks. The sapphire used for the stylus on record turntables is another example.

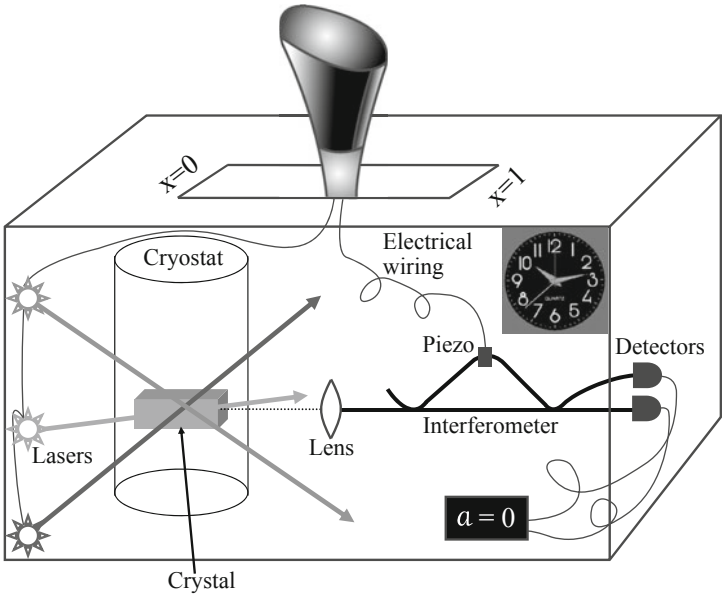

Fig. 2.2 Inside Bell's boxes, Alice and Bob discover a complicated set of physical equipment. But it is not just by observing locally the contents of the two boxes that Alice and Bob could understand how Bell's game works, because the game can produce correlations with no local explanation

identical. Once again, the secret seems to be contained within the tiny crystal at the center of the setup.

In the end, this examination of the boxes is hardly conclusive. And that is precisely the message of this chapter. Simply investigating the way the boxes are put together and the way they work, even in great detail, we will never find a satisfactory explanation. In any case, we already know that there can be no local explanation for winning Bell's game, so it will certainly not be by looking locally in each of the two boxes that we will find an explanation! So let us step back for a moment. At the end of the day, all this complicated equipment serves only to produce a binary result each time the joystick is pushed to the left or to the right. Therefore, even if the mechanism is very complicated, it seems that all it can do is produce one of the four programs described earlier (see p. 16). After all, what else could it do? Once again, there are only two possible positions for the joystick and just one binary result. Using several lasers, a cryostat, and photon detectors seems excessive to carry out such a simplistic program! But this setup does more, since it allows us to win Bell's game.

A pre-quantum physicist might have spent a great deal of time studying these devices and understood nothing at all, so the reader should not be embarrassed if it is not clear what is going on inside them. We shall see the solution in Chap. 6, but for the moment, let us just note that the core elements in the boxes are crystals, and that these crystals are entangled.[11] But what does that mean? At present, 'entanglement' is just a word we use to name the concept of quantum physics that allows us to win Bell's game. So patience!

In conclusion, the exact content of the boxes is not so important. The only important thing is that physicists know—in principle—how to build boxes that allow Alice and Bob to win Bell's game more often than 3 times out of 4, and that the essential ingredient goes by the name of *entanglement*. The simple fact that it is possible to win Bell's game is a significant conclusion, a fact as blindingly clear as a photograph of the Earth floating in empty space: the Earth is round, and quantum physics predicts nonlocal correlations.

[11] For the specialist, we should say that it is not the whole of Alice's crystal that is entangled with Bob's. Rather, each crystal contains several billion rare earth ions. A few collective excitations of these ions in Alice's crystal are entangled with similar excitations of the ions in Bob's crystal. (Christoph Clausen, Imam Usmani, Félix Bussières, Nicolas Sangouard, Mikael Afzelius, Hugues de Riedmatten, and Nicolas Gisin: *Quantum storage of photonic entanglement in a crystal*, Nature **469**, 508–511, January 2011.)

3
Nonlocality and True Randomness

We have seen that it is easy to obtain a score of 3 in Bell's game. For example, we need only agree beforehand to produce the same result each time. But we have also seen that it is impossible to specify any local strategy that could be applied independently by Alice and Bob and that would allow them to win more often than 3 times out of 4. This was the main conclusion of Chap. 2.

But if two players were to win Bell's game in the sense of obtaining a score greater than 3 out of 4, what must we conclude in this case? The obvious conclusion seems to be twofold: either they influence one another in some subtle way, or else they are accomplished cheats. But let us suppose we can eliminate these two options. The second possible conclusion is that there may be an error in the argument presented in Chap. 2. Many physicists and philosophers have spent years looking into this. Why not try yourself for a few moments? Remember that one should never accept an argument on authority. Everyone has the right, and the duty, to check scientific reasoning for himself. This is all the more important here in that the argument showing that Bell's game cannot be won without communication is simple and limpid. Once again, each of the two players Alice and Bob can only choose one of four possible strategies. There are thus only $4 \times 4 = 16$ possible combinations of strategies and none of them provides a way of winning more often than 3 times out of 4 (see Table 2.1 in Chap. 2). Go through the reasoning again. Try explaining it to a friend.

There is good reason to be convinced of the cogency of this argument. It is perfectly sound and has been checked by thousands of physicists, philosophers, mathematicians, and specialists in computing and information theory. But then, why raise the issue of players winning more often than 3 times out of 4, since that seems impossible? This is the burning question. The argument is so simple that, without quantum physics, nobody would be the least bit interested. It would just be one more obvious fact among stacks of other totally uninteresting obvious facts of no particular relevance to anything. The only good reason for examining this question is that today physics can in fact win this game, and it can do so without the players either communicating or cheating.

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A Nonlocal Whole

So let us return to our question: what conclusion should we draw from a score systematically greater than 3 in Bell's game? The only possibility is that Alice and Bob's boxes, although separated in space, are not logically separated. Despite the distance between them, we cannot describe Alice's box on one side and Bob's on the other as separate entities. Put another way, we cannot just say what Alice's box is doing on the one hand and what Bob's is doing on the other. It is as though, despite the distance between them, the two boxes were acting as a single entity, that is, an entity that cannot be logically separated into two parts. In short, a nonlocal whole.

But what is a nonlocal whole? Does that really help you to understand? Probably not, unless you are truly brilliant! Here the word 'nonlocal' simply describes something that cannot be described as two independent and well-localised parts. Of course, Alice and Bob and their two boxes are well localised, just like any normal people or boxes. We may surround them by reinforced concrete walls and lead casing, or anything else for that matter, but it is impossible to describe their behaviour as being like this for Alice's box and like that for Bob's box. Indeed, if each had a behaviour and hence a strategy of its own, it would be impossible for them to win Bell's game. And this is so even if this behaviour and these strategies were discussed and coordinated before the boxes were moved apart in space.

And so we reach a quite remarkable conclusion, and not easy to swallow. If Alice and Bob achieve a score greater than 3, then we are forced to admit that, despite the distance between them and the possibility of identifying the two players, the results of their games cannot be produced locally, one by Alice's box and the other by Bob's. *These results are produced in a nonlocal manner*. It is as though Alice's box 'knew' what Bob's was doing, and conversely.

Box 6. A Nonlocal Calculation. Winning Bell's game means that Alice's and Bob's results 'relate' to each other in such a way as to satisfy the equation $a + b = x \times y$ more often than 3 times out of 4. Hence, the product $x \times y$ is correctly estimated more often than would be possible locally, even though the inputs x and y exist jointly nowhere, i.e., x is only known to Alice and her box and y to Bob and his box. Here surfaces an idea for a quite astonishing calculator, the quantum computer, even though the story of this computer is still long and goes well beyond the scope of this book (and in fact one should speak rather of a quantum processor than a genuine all-purpose computer).

Telepathy and True Twins

At this juncture, some readers may think of telepathy, or perhaps twins who, when separated, make the same decisions or suffer the same ills. But such escape routes would be misleading.

Let us begin with the twins. What characterises twins is the fact that they share the same set of genes. They carry the same genetic blueprint and will thus look alike, indeed often quite indistinguishable. This is like local Alices and Bobs who likewise 'carry' strategies analogous to genetic instructions. But we have seen that, whatever strategies are 'carried' by Alice and Bob or memorised by their boxes, they will never be able to win Bell's game. And likewise, two perfectly identical twins, even if they have been subject to precisely the same environmental influences throughout their existence, will never be able to win Bell's game. Hence, the twin analogy is perfect for understanding local correlations, but it provides no way whatever of understanding how one might go about winning Bell's game. On the contrary, even ideal twins cannot win Bell's game.[1]

So what about telepathy? If it existed, it would allow two people to communicate at a distance through thought. The great difference with Bell's game is that, to win this game, there is no need to communicate. It suffices for the results to be produced randomly, but in a coordinated way. Each of Alice and Bob's boxes must in some sense 'know' what the other is doing, but the players themselves cannot used this 'knowledge' to transmit information. Therefore, to win Bell's game, the players are not using telepathy, even if we could imagine that the boxes are somehow telepathic.

Personally, I don't much like the idea that the boxes are capable of telepathy because I don't see what that would add to our understanding. I have the impression that it amounts to replacing one word (nonlocal) by another (telepathy). But if you feel that this terminology helps you to grasp the idea, then why not, provided that you bear in mind that these are not people who can engage in telepathy, just boxes or the crystals sitting at their center. Furthermore, this terminology is misleading, because the notion of telepathy involves an emitter and a receiver. But we shall see later that this idea is hardly possible. In addition, in Bell's game and associated experiments, the symmetry between Alice and Bob is perfect. There is nothing to distinguish a possible emitter or receiver.

Coordination is Not Communication

The idea of a nonlocal whole immediately summons up a notion of instantaneous communication. Recall Newton's reaction to the nonlocality in his universal theory of gravitation. Indeed, if Alice and Bob's boxes win Bell's

[1] In this sense, the image of twin photons often used to speak of entangled pairs of photons, which *can* in fact be used to win Bell's game, is extremely misleading.

game, it is because they coordinate after their joysticks have been pushed to the left or the right. But since they are separated by a large distance, these boxes must be able to coordinate at a distance. This is what Einstein referred to as a spooky action at a distance, an expression that clearly emphasises the master's distaste for such a thing. But lo and behold, today, many experiments contradict Einstein's intuition and corroborate quantum theory: nature is indeed capable of coordinating two remote boxes.

However, coordination does not mean communication. But how can one coordinate without communicating? As human beings, we would certainly be incapable of such a feat, and we thus have great difficulty imagining how it could be done. In fact, in order to coordinate without communication, the boxes must produce their results at random. To see this, we start by assuming the opposite, that is, that the boxes produce predetermined results. We shall then show that this would allow Alice and Bob to communicate without transmission. But since such communication without transmission is impossible (see Box 5 on p. 22), we may conclude that any boxes able to win Bell's game cannot produce predetermined results.

To get a better hold on the question, we begin by imagining the simple case where Alice's box always produces $a = 0$ and Bob chooses $y = 1$. If Bob obtains the result $b = 0$, he knows that $a = b$ and can deduce that Alice probably chose $x = 0$. Otherwise, if he obtains $b = 1$, he knows that $a \neq b$ and can deduce that Alice probably chose $x = 1$. Indeed, it would only be in this way that they would obtain a point in Bell's game. Box 7 shows that this important conclusion remains true whatever the relation that determines Alice's result as a consequence of her choice.

From the above, what we should understand is that, if Alice's box produces the result a in a deterministic way (following one of the 4 programs presented on p. 16) and Bob knows this way, then Bob can deduce Alice's choice from the result of his box. Hence, according to this hypothesis, Bob would be able to read Alice's thoughts from a distance. Indeed, each time they obtain a point, Bob can correctly guess Alice's choice. If they win Bell's game, this sort of communication would be commonplace.

Such communication would be almost instantaneous because the transmission time does not depend on the distance separating Alice and Bob. In particular, the speed of the communication could be faster than the speed of light, but the speed of light does not enter the reasoning here, because by moving Alice and Bob further and further apart, one could exceed any speed. Even more important, one would have a non-physical form of communication, since no transmission would be required to carry it between Alice's box and Bob's box. But such communication without transmission is impossible (see Box 5 on p. 22).

To sum up then, if Alice and Bob's boxes are able to coordinate at a distance, but in such a way that this cannot be used by Alice and Bob to communicate, then Alice's result cannot be produced in a deterministic way. It must necessarily be produced randomly, through some nonlocal chance-like process.

Box 7. Determinism Would Imply Communication Without Transmission. According to the hypothesis of determinism, there is some relation that determines the result produced by each box as a consequence of the direction in which the joystick is pushed. But any deterministic relation between the direction of Alice's joystick and its result would allow Bob to read Alice's choices from a distance and hence to achieve communication without transmission. And since such communication is impossible, determinism is also impossible. To convince ourselves of this conclusion, we shall examine a second example.

Let us imagine that, if Alice's joystick is pushed to the left, her box produces the result $a = 0$ and if it is pushed to the right, then the result is $a = 1$. This corresponds to strategy 3 in Chap. 2, i.e., $a = x$ (see p. 16). In this case, if Bob pushes his joystick to the left, hence $y = 0$, he can deduce the direction in which Alice has pushed her joystick from the result produced by his box. So for example, if his result is $b = 0$, Bob knows that Alice probably pushed her joystick to the left, because this would be the only case in which they would get a point in Bell's game. Indeed, for $y = 0$, we must have $a = b$ to win a point, and as Bob observes $b = 0$, he deduces that $a = 0$, but this result is only possible in our example if $x = 0$, and hence if Alice pushed her joystick to the left.

To convince ourselves that this conclusion remains valid whatever relation determines the result a as a consequence of the choice x, it suffices to note that, among the four variables in the equation $a + b = x \times y$, Bob knows two, his choice y and his result b. If in addition he knows the relation $a = \text{function}(x)$, then Bob can calculate the choice x made by Alice. For example, if $a = x$, then the relation $a + b = x \times y$ can be rewritten as $x + b = x \times y$, and then if Bob chooses $y = 0$, we have $x = b$, i.e., the result produced by Bob's box is equal to Alice's choice x.

The relation which determines Alice's result as a consequence of the direction in which she pushes her joystick may change from one minute to the next, but at each minute this relation would be fixed, in fact, pre-established a long time beforehand. If such were the case, nothing could stop Bob from knowing this relation for each minute. But for each relation, Bob can guess with a high probability the direction in which Alice pushed her joystick, whereupon it is just as though he could read Alice's thoughts at a distance. We would have communication without transmission.

Nonlocal Randomness

We have just seen why Alice and Bob's results must be produced by chance, but that this randomness is not independent in Alice's box and Bob's box, because in fact *the same chance-like events occur in Alice's box and in Bob's box*. So this is a fascinating thing! Chance is already a fascinating concept, but here, in addition, the same chance-like event is manifested at two widely separated points. This explanation goes dead against our common sense, but

it is unavoidable. If you are having difficulty in swallowing this, rest assured that many physicists find themselves in the same situation, including Albert Einstein, who never believed that it might be possible to win Bell's game.

We shall thus turn all our attention to this 'nonlocal randomness' in Chap. 5 and then explain the experiments which allow us to win Bell's game in Chap. 6. We shall look closely at the relevance of these experiments in Chap. 9 to see whether there might not be some loophole that would allow us to save the notion of locality.

But before ending this chapter, let us come back to our 'explanation'. I have put scare quotes on this word because we have reached a point where we should ask ourselves what we mean by an explanation, or what we require of it. An explanation is basically a story which tells us about the phenomenon to be explained. Notwithstanding, some readers may object, and with reason, that just talking about nonlocal randomness hardly comes up to the standards of an explanation. But the conclusion is nevertheless unavoidable: we can relate no story that takes place locally in space and continuously in time that would tell us how to win Bell's game.

Remember Newton's contemporaries when they were asked to swallow the 'explanation' that everyone falls toward the center of the Earth. Is that an explanation? Well, yes and no. The explanation by gravitation has the great advantage of taking place in our time (we fall) and in our space (toward the Earth), but it does leave open the question of how our body 'knows' where the Earth is, even when our eyes are closed.

The explanation by nonlocal randomness may be less satisfying than the explanation by free fall. But the main point is that there will be no explanation based solely on localised entities. Winning Bell's game amounts to demonstrating that *nature is nonlocal*.

So should we perhaps abandon any attempt to produce an explanation? Certainly not! We must just accept to tell a story that involves nonlocal features like nonlocal randomness, an irreducible randomness that can manifest itself in several widely separated places without propagating from one point of space to the next.[2] Nonlocality forces us to extend the conceptual toolbox we use to talk about nature's inner workings.

[2] I do not intend to assert that the explanation in terms of nonlocal randomness is complete and definitive. However, I would say that scientists will always try to find explanations, and that any explanation for this must necessarily be nonlocal. The explanation that history will eventually adopt will be the one that allows us to go beyond the physics of today and leads us to the discovery of a new physics that incorporates quantum physics as an approximation. This new physics will still allow us to win Bell's game, otherwise it will not be in agreement with experimental results. For this reason, it will also be nonlocal.

To help you here, imagine a kind of nonlocal 'dice' which can be thrown by pushing either of the two joysticks. This nonlocal dice produces the result a for Alice as soon as she has pushed her joystick in the direction x and a result b for Bob as soon as he pushes his joystick in the direction y. Both a and b are random, but with the guarantee that they 'attract' to each other in such a way that they often satisfy the goal of Bell's game, that is, they often satisfy the equation $a + b = x \times y$.

From the moment we accept that the world is not deterministic, and therefore that irreducible randomness really exists, we must also accept on the one hand that this randomness is not necessarily governed by the same laws as those presiding over classical probabilities,[3] and on the other that there is nothing in principle to forbid it from manifesting itself in several places at the same time, provided that the effect cannot be used to communicate.

True Randomness

We have just seen that there is only one way to avoid a situation where winning Bell's game would lead to the possibility of communicating at arbitrarily high speeds: Alice's box must not produce its results each minute as a consequence of predetermined relations, but must generate them in a truly random manner. Only the hypothesis of irreducible randomness can prevent Bob from knowing the relation between Alice's choice and her result. If this were not true randomness, Bob would end up finding the relation, and so would the world of physics.

We must therefore give up the idea that Alice's box produces a result locally. It is the two boxes in unison that produce a pair of results globally, even if from the point of view of each of the two partners, their own result is chance-like.

The notion of true randomness deserves special attention. The typical example of chance-like events is provided by the game of heads-or-tails with a

[3] In classical physics, the result of any measurement is predetermined. In a sense, it is written into the physical state of the system being measured. Probabilities come in only through our ignorance of the exact physical state. This ignorance compels the scientist to resort to statistical methods and probabilistic calculations obeying Kolmogorov's axioms. In quantum physics, the result of a measurement is not predetermined, even if the state of the system is known perfectly. Only the propensity of such and such a result to manifest itself is written into the physical state of the system being measured. These propensities do not obey the same rules and do not satisfy Kolmogorov's axioms. But note that certain results in quantum physics are nevertheless predetermined. The structure of the mathematical theory of quantum physics (Hilbert space) is such that, for states allowing no ignorance, the so-called pure states, the set of all predetermined results uniquely characterises the propensity of all other possible results. In this sense, the propensities of quantum physics are a logical generalisation of classical determinism. (N. Gisin: *Propensities in a non-deterministic physics*, Synthese **89**, 287–297, 1991; see also arXiv:1401.0419.)

coin, or the throwing of a dice. In both cases, it is the complexity of microphenomena such as the impacts of air molecules on the coin or the roughness of the surface on which the dice bounces which makes it impossible in practice to predict the result. But there is nothing intrinsic about this impossibility, which is merely the consequence of many otherwise insignificant causes that work together to build up the final result. If we were to follow the details of the dice's motion with sufficient care and adequate means of calculation, given the initial conditions of the throw, the air molecules, and the surface on which the dice bounces and eventually comes to rest, then we could surely predict the face it would present to us. This is not therefore a case of true randomness.

Another example will better bring out the distinction we are making. To carry out numerical simulations, engineers often use so-called pseudo-random numbers. Many problems can be analysed in this way. Think, for example, of the development of an aircraft. Rather than manufacture dozens of prototypes and test them one by one, engineers simulate these prototypes on big computers. To simulate flight conditions, which can change all the time depending on the wind and all kinds of unpredictable influences, engineers use pseudo-random numbers in their simulation programs. These numbers are produced by computers, which are themselves deterministic machines in which chance plays no role whatever. So these numbers are not in fact produced randomly, but just like the results when we throw a dice, whence the qualifier 'pseudo'. The relation between one pseudo-random number and the next is predetermined, but it is complicated enough to ensure that it cannot be guessed easily. At first glance, one might think that this would be sufficient, and that there would be no real difference between the pseudo-random numbers produced by a computer and numbers produced by true randomness. But this would not be correct. There are aircraft prototypes that behave perfectly when they are simulated with these pseudo-random numbers, but which in reality fly rather badly.[4] These cases are rare, but they nevertheless exist, however ingenious the program producing the pseudo-random numbers. On the other hand, for numbers produced in a truly chance-like manner, there are no such pathological cases. There is thus a real difference between apparently chance-like events such as dice throws and truly chance-like events of the kind required to win Bell's game without allowing communication. In addition, we see that the existence of true randomness is a useful resource for society. We shall return to this in Chap. 7.

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[4] Ferrenberg, A.M., Landau, D.P., Wong, Y.J.: *Monte Carlo simulations: Hidden errors from 'good' random number generators*, Phys. Rev. Lett. **69**, 3382 (1992); Ossola, G., Sokal, A.D.: *Systematic errors due to linear congruential random-number generators with the Swendsen--Wang algorithm: A warning*, Phys. Rev. E **70**, 027701 (2004).

True Randomness Allows Nonlocality Without Communication

In conclusion, winning Bell's game without communication necessarily implies that Alice and Bob's boxes produce results in a truly chance-like way. This randomness is fundamental and cannot be reduced to a complex deterministic mechanism. This means that nature is capable of pure acts of creation!

Rather than asserting as Einstein did that God does not play dice, let us ask rather why he plays dice.[5] The answer is that, in this way, nature can be nonlocal without that leading to the possibility of communication without transmission. Indeed, once we accept that nature can produce truly chance-like events, there is no reason to restrict the manifestation of this randomness to a single well-localised place. True randomness can manifest itself in several places at the same time. Since such nonlocal randomness cannot be used to communicate, there is no reason to restrict nature.

We have thus found that two apparently highly disparate concepts, chance and locality, are in fact closely related. Without true randomness, locality is necessary to avoid the possibility of communication without transmission. We must thus keep in mind that true randomness exists and that it can manifest itself in a nonlocal way. We must simply get used to this idea of a randomness that can manifest itself in several places, that is, a nonlocal randomness which coordinates results occurring in two widely separated places. We must integrate into our intuitions the idea that nonlocal randomness cannot be used to communicate. It is as though Alice and Bob could only 'hear' noise, as produced by the peculiar 'telephone', that is, noise which cannot be used to communicate but which does allow us to win Bell's game.

[5] Popescu, S., Rohrlich, D.: *Nonlocality as an axiom*, Found. Phys. **24**, 379 (1994).

4
Impossibility of Quantum Cloning

Nonlocality without communication has other surprising consequences. One example concerns the issue of quantum cloning. In our context, this amounts to trying to produce a copy of Bob's box. This relatively simple example lies at the heart of quantum cryptography, and also teleportation, as we shall see in Chaps. 7 and 8. It is thus worth taking a look.

Animal cloning has become commonplace. No doubt human cloning is already within reach and will be achieved before the end of this century. Beyond the quite legitimate emotional reaction and the scandal that this will stir up, let us enquire whether cloning is possible in the quantum world. In other words, is it possible to copy a physical system belonging to the world of atoms and photons? Can physicists produce a clone, i.e., a perfect copy, of Alice's box or Bob's box?

Let us be more precise. It would be absurd to 'copy' an electron, since all electrons are strictly identical. When we talk of copying a book, we are not trying to produce another book of the same format and with the same number of pages. A copy must contain exactly the same information, hence the same text and the same images. The clone of an electron will thus have to carry the same 'information' as the original, and hence have the same average velocity and the same indeterminacy[1] in this velocity, and likewise for all other physical quantities. Only the average position will be different, so that we can have the original over here and the clone over there.

In this chapter, we shall ask whether it is in fact possible to clone Bob's box. We have already seen that the core elements of these boxes are crystals carrying the quantum characteristic we called entanglement. So cloning Bob's box amounts at the end of the day to cloning these quantum entities along with their quantum features.

[1] For historical reasons, physicists often speak about quantum uncertainty. But since (un)certainty refers rather to an observer than to the physical system, we prefer today to speak of quantum indeterminacy (see Box 8 on p. 38).

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Box 8. Heisenberg's Uncertainty Relations. Werner Heisenberg was one of the main founders of quantum physics. In particular, he is remembered for his statement of the uncertainty principle, according to which if we measure the position of a particle with some accuracy, we necessarily disturb its velocity. And conversely, if we accurately measure the velocity of a particle, we necessarily disturb its position. We can thus never know the position and the velocity of a particle at the same time and with great accuracy. Modern quantum physics has incorporated this principle in the fact that particles simply never simultaneously have a precisely determined position and a precisely determined velocity. As a consequence, we prefer to speak of indeterminacy. We shall keep the word 'uncertainty' for Heisenberg's principle, but we shall avoid qualifying physical quantities as uncertain. We shall say that they are indeterminate. Likewise, uncertainty becomes indeterminacy.

Quantum Cloning Would Allow Impossible Communication

The impossibility of cloning a quantum system is essential for applications such as quantum cryptography and quantum teleportation, to be discussed later. To prove this impossibility, we argue by *reductio ad absurdum*, that is, we begin by assuming that it is in fact possible to clone quantum systems, and we then deduce something absurd, in this case, communication without transmission. We shall thus be able to conclude that, since communication without transmission is impossible, quantum cloning must also be ruled out.

Imagine that Bob succeeds in cloning his box. More precisely, imagine that Bob succeeds in cloning the crystal at the heart of his box, recalling that the rest of his box is merely a complex mechanism which would not be difficult to reproduce. So here he is with two boxes in front of him, let's say one on the left and one on the right. Each of these two boxes has a joystick that he can push to the left or the right, and one second later, each box will then produce a result. If these two boxes really are clones, the results they produce will both be correlated with the result from Alice's box, in such a way that each box will win its Bell game with Alice. However, Bob can decide not to choose, but rather to try both choices at the same time. That is, he pushes the joystick of the left-hand box to the left and that of the right-hand box to the right. We shall now explain how Bob can deduce from his two results the choice that Alice has made at a great distance from him.

Let us begin with the case where Bob's two results are identical, hence either twice 0 or twice 1. In this case, Alice probably chose $x = 0$. Indeed, if Alice had chosen $x = 1$, the result from Bob's right-hand box would have to differ from Alice's result [because $(x, y) = (1, 1) \Rightarrow a \neq b$], whereas the result from his left-hand box would have to be the same as Alice's [because $(x, y) = (1, 0) \Rightarrow a = b$]. On the other hand, if Bob's two results are different,

then Alice probably chose $x = 1$. Box 9 sums up this little argument using elementary binary arithmetic.

Box 9. No-Cloning Theorem. Let b_{left} and b_{right} denote the results produced by Bob's two boxes, the one on the left and the one on the right, respectively. Winning Bell's game implies that the following two relations are often satisfied: $a + b_{\text{left}} = x \times y_{\text{left}}$ and $a + b_{\text{right}} = x \times y_{\text{right}}$. Adding these together, we obtain

$$a + b_{\text{left}} + a + b_{\text{right}} = x \times y_{\text{left}} + x \times y_{\text{right}} .$$

Now recall that all these symbols represent bits (0 or 1) and that addition is modulo 2 so that the result is always another bit. Hence, $a + a = 0$. Recall also that Bob pushes the joystick of the left-hand box to the left, whence $y_{\text{left}} = 0$, and the joystick of the right-hand box to the right, whence $y_{\text{right}} = 1$. Finally, we obtain $b_{\text{left}} + b_{\text{right}} = x$. Bob can thus guess Alice's choice x with high probability simply by adding together his two results.

Hence, if Bob were able to clone his box, he could guess Alice's choice with high probability, and this despite the great distance that may separate them. There would thus be communication without transmission, and at an arbitrary speed. Some may observe that Bob could make a mistake when he guesses Alice's choice, since Alice and Bob do not achieve a score of 4, but only a score that is significantly greater than 3. Effectively, Bob may sometimes be mistaken. However, the fact that he guesses correctly much more often than 1 out of 2 times is enough to make communication possible.[2] This communication would be a bit noisy and they would have to repeat many times (with Alice making exactly the same choice each time) so that Bob could end up guessing Alice's choice almost with complete certainty. In fact, this is already what happens with all digital communication. Internet and other communication protocols cut our messages into little pieces which they send to the receiver, and since there is always some small probability of error, the message is sent back several times until the probability of any error remaining can be considered negligible.

In conclusion, the possibility of winning Bell's game implies the impossibility of cloning quantum systems. Physicists call this the no-cloning theorem. It is an extremely important result in quantum physics. It is very easy to prove mathematically, but we have seen that this theorem is also very straightforwardly deduced from the existence of nonlocality without communication, and this once again emphasises the importance of this concept.

[2] It can be shown that Bob guesses Alice's choice correctly more often than 1 time out of 2 precisely if Alice and Bob win Bell's game more often than 3 times out of 4.

How is it Possible to Clone DNA?

We may wonder how it is possible to clone animals if we cannot clone quantum systems. For surely the biological macromolecule known as DNA is itself a quantum system? It is remarkable that it was precisely by asking this question that Nobel prize winning physicist Eugene Wigner was first to raise the issue of quantum cloning.[3] In fact, he also concluded the impossibility of cloning in biology, but that was a mistake. DNA is indeed a quantum system (at least, it seems highly likely to be, and although that has never been demonstrated experimentally, no physicist would doubt it). However, the genetic information in DNA is encoded using only a very small fraction of the possibilities allowed by quantum physics, and there is no fundamental obstacle to cloning this small amount of information.[4] As a general question, it is interesting to inquire about the role of quantum physics in biology, and this is a topical research theme.

Digression: Approximate Cloning

To end this chapter, allow me to make a few remarks that will not be essential to the main theme, but which could be of interest to scientific readers.

So let us note, but this time without proof, that quantum theory does allow approximate cloning, something like poor copies, and that the best possible cloning is precisely characterised by the fact that it is just poor enough to ensure that there can be no communication without transmission.[5]

The no-cloning theorem is closely tied to many aspects of quantum theory. In particular, as already mentioned, it is essential to applications like quantum cryptography (see Chap. 7) and quantum teleportation (see Chap. 8). It is also vital if Heisenberg's famous uncertainty relations are to have any meaning (see Box 8 on p. 38). Indeed, if it were possible to perfectly clone a quantum system, we could then measure the position, for example, on the original, and the velocity on the clone. We would thus obtain the position and velocity of

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[3] Wigner, E.P.: The probability of the existence of a self-reproducing unit. In: *The Logic of Personal Knowledge: Essays Presented to Michael Polanyi on his Seventieth Birthday*, Routledge and Kegan Paul (1961). Reprinted in Wigner, E.P.: *Symmetries and Reflections*, Indiana University Press (1967) and in *The Collected Works of Eugene Paul Wigner*, Springer-Verlag (1997), Part A, Vol. III.

[4] It is as though information were encoded in the position of an electron, without worrying about its velocity. In this case, the position can be copied, and while this would perturb the electron's velocity, that would be of no importance, since the velocity is not being used to carry information.

[5] Gisin, N.: *Quantum cloning without signalling*, Physics Letters A **242**, 1–3 (1998).

a particle at the same time, something that is ruled out by the uncertainty principle.[6]

Another important consequence of the no-cloning theorem is that the stimulated emission that underlies the production of laser light is not possible without also having spontaneous emission. Otherwise, one could use the stimulated emission to perfectly clone the state of a photon (e.g., its polarisation). Once again, the ratio between stimulated emission and spontaneous emission is precisely at the limit of optimal cloning compatible with nonlocality without communication.[7] Everything fits together very nicely. Quantum theory is remarkably consistent and elegant. Einstein was the first to describe the ratio between stimulated emission and spontaneous emission. He would have been astonished to learn that his formula followed precisely from the notion of nonlocality which he so detested.

Finally, one last remark regarding the relationship between cloning and nonlocality. We have seen that the impossibility of communication without transmission imposes a limit on the quality of cloning Bob's box into two boxes. What happens when we replace Bell's game by a game (or inequality) in which Bob has access to more possibilities? Suppose, for example, that the joystick can be pushed in n different directions. In this case, the impossibility of communication without transmission imposes a limit on the cloning of Bob's box into n clones, and we still obtain the optimal quantum cloning limit. A consequence is that, to demonstrate nonlocality, Bob, and also Alice, must have more available choices than boxes, so they must have to make a real choice. They cannot just make all the choices in parallel.[8] Here we begin to glimpse the importance of free will, or put more prosaically, the importance of Alice and Bob's being able to make free choices independently of one another. Without independent choices, there would be no nonlocality.

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[6] This is a bit of a simplification as it could be that the statistics of position measurements on the original system and the statistics of velocity measurements on the clone do satisfy the Heisenberg uncertainty relation. The truth is that the uncertainty relation as formulated historically by Heisenberg is vague, if not wrong [see, e.g., M. Ozawa: Phys. Rev. A **67**, 042105 (2003)]. One way of formulating it precisely is the no-cloning theorem and its optimal quantum approximation [see, e.g., C. Branciard: Proc. Natl. Acad. Sci. USA **110**, 6742–6747 (2013)].

[7] Simon, C., Weihs, G., Zeilinger, A.: *Quantum cloning and signaling*, Acta Phys. Slov. **49**, 755–760 (1999).

[8] Terhal, B.M., Doherty, A.C., Schwab, D.: *Local hidden variable theories for quantum states*, Phys. Rev. Lett. **90**, 157903 (2003).

5
Quantum Entanglement

According to quantum physics, the explanation for the possibility of winning Bell's game, in the sense of obtaining a score greater than 3, is entanglement. Erwin Schrödinger, one of the fathers of quantum physics, was the first to note that entanglement is not just one feature of quantum physics among others, but its main characteristic:[1]

> Entanglement is not one, but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.

In this chapter, we shall present this remarkable property of the world of atoms and photons.[2]

Quantum Holism

Roughly speaking, the strange theory of quantum physics tells us that it is possible and even commonplace for two widely separated objects in space to form in reality a single entity! And that's entanglement. If we then prod one of the two parts, both will quiver. First of all, note that when we 'prod', that is, when we carry out a measurement on a quantum entity, it will produce a response—a reaction—completely by chance, just one result among a certain range of possibilities, with a well-defined probability that quantum theory predicts with perfect accuracy. Since this is a chance event, one cannot use the fact that the entangled entity reacts as a whole to send any information. Indeed, the receiver would merely receive noise, a purely random palpitation. Once again we discover the importance of true randomness. But then, you may say, if we don't prod the first object, the second won't quiver. I can thus send information simply by deciding whether or not to prod the first object.

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[1] Schrödinger, E.: *Discussion of probability relations between separated systems*, Proceedings of the Cambridge Philosophical Society **31**, 55 (1935).

[2] For a more detailed discussion, see Scarani, V.: "Quantum Physics, A First Encounter", Oxford Univ. Press 2006.

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But the problem is this: how would one know whether the second object had quivered? To find out, one would have to carry out a measurement, and this would make it quiver. In short, no matter how counterintuitive it may be, the idea that two entangled objects in fact constitute a single whole cannot be ruled out by simple arguments.

In theory, any object can be entangled, but in practice, physicists have demonstrated the entanglement of atoms, photons, and some elementary particles. The biggest objects that have been entangled are crystals like the ones in the boxes of Bell's game. Entanglement is much the same whatever the objects that get entangled. We shall illustrate this almost magical property of the quantum world using electrons, the tiny particles that carry electric current.

Quantum Indeterminacy

We begin with an example. An electron may occur in a state in which its position is indeterminate. So it simply does not have a precise position, rather like a cloud. Naturally, a cloud has an average position (the position of its center of mass, as physicists would say), and the electron also has an average position. But, and it is a very significant difference with the cloud, an electron is not made up of a multitude of water droplets, nor of any other kind of droplets. The electron is indivisible. And yet, while it is indivisible, it does not have a position, but only a cloud of potential positions. If despite this we proceed to measure its position, the electron will duly reply: "Here I am!" But this response was produced during the measurement, by pure chance. The electron had no position but, during the measurement process, it was compelled to respond to a question that had no predetermined answer: quantum randomness is true, irreducible randomness.

Formally, this indeterminacy is expressed by means of what is known as the superposition principle. If an electron can be here or a meter to the right of here, then this electron can also be in a superposition state of 'here' and 'a meter to the right of here', that is, both here *and* a meter to the right of here. In this example, the electron is delocalised in two places at once. It may feel what happens here, for example, in one of Young's slits, and feel what happens one meter to the right of here, in the other Young slit. [The name of Thomas Young (1773–1829) is associated with a famous experiment in which a particle passes through two neighbouring slits at the same time.] Therefore it is indeed both here *and* a meter to the right of here. However, if we measure its position, we obtain either the result 'here' or the result 'a meter from here' in a completely random manner.

Quantum Entanglement

We have just seen that an electron may not have a position. Likewise, each of two electrons may very well not have a position. But due to entanglement, the distance between the two electrons may nevertheless be perfectly definite. That would mean that, whenever we measure the positions of the two electrons, we obtain two results, each completely random, but whose difference is always exactly the same! Relative to their average positions, the two electrons will always produce the same result, even though this result is truly chance-like. So if one of the electrons is measured a little to the right of its average position, then the other will also be measured a little to the right, and by precisely the same small amount as the first, that is, precisely the same distance from its center of mass. And this remains true even if the two electrons are very far removed from one another.

The position of one electron relative to the other is thus well defined, even though the position of each electron is not. In general, entangled quantum systems can be in a well determined state, even though each system is itself in an indeterminate state. When measurements are carried out on two entangled systems, the results are governed by chance, but by *the same* chance! Quantum randomness is nonlocal.

Entanglement can also be defined as the ability of quantum systems to produce the same result when we measure the same physical quantity on each of them. It is described by applying the principle of superposition simultaneously to several systems. For example, two electrons can be 'one here and the other there', or they can be 'the first a meter to the right of here and the other a meter to the right of there'. According to the principle of superposition, these two electrons can also be in the state 'one here and the other there' superposed on 'the first a meter to the right of here and the other a meter to the right of there'. This is an entangled state. But entanglement involves much more than just the superposition principle, for it is entanglement that introduces nonlocal correlations into physics. For example, in the above entangled state, no electron has a predetermined position, but if a measurement of the position of the first electron produces the result 'here', the position of the other electron is immediately determined as 'there', even without having to measure the position of the second electron.

How is this Possible?

How can two electrons have a well-defined relative position without either having a definite position? In the world of everyday objects, this would be impossible. It is thus natural to suspect that quantum physics is not giving us

a complete description of the positions of the electrons, and that a more complete theory would describe the electrons as always having a well-defined but hidden position. This is the intuition that underlies the notion of local hidden variables. These are local because each electron has its position independently of the other electrons.

Nevertheless, this hypothesis of hidden positions is not without raising its own difficulties. Indeed, the position of an electron is not the only measurable variable. For we may measure its velocity, which is also indeterminate. An electron certainly has an average velocity, but the velocity produced during a measurement is produced by chance among a whole range of possible values, just like the position produced by a position measurement. And once again entanglement allows two electrons each to have no definite velocity while both can have exactly the same velocity, and this remains true even if the electrons are very far removed from one another.

Entanglement actually goes one step further. The two electrons may each have neither position nor velocity but be entangled in such a way that the differences between their positions and between their velocities are perfectly determined. If there are hidden positions, there must therefore also be hidden velocities. But that contradicts Heisenberg's uncertainty principle which is a key part of the quantum formalism (see Box 8 on p. 38). Werner Heisenberg, his mentor Niels Bohr, and their friends reacted strongly against the assumption of hidden positions and velocities, which were referred to as local hidden variables. On the other hand, Erwin Schrödinger, Louis de Broglie, and Albert Einstein endorsed the hidden variable hypothesis as being more natural than the hypothesis of entanglement, which implies a true randomness that can manifest itself simultaneously in several places at once.

At the time, and right through from 1935 to 1964, nobody had found anything like the argument put forward by John Bell which we discussed in Chap. 2. Hence, there was no physical experiment that could decide the debate through a genuine experimental test, for example, by answering the question: is it possible to win Bell's game more often than 3 times out of 4? If there were local hidden variables, quantum systems could not win Bell's game. The local hidden variables (like the twins' genes) would play the role of programs that would determine in a local manner, at Alice's location and at Bob's, the results produced by the two boxes. But we have seen that, if the results are determined locally, then Alice and Bob cannot win Bell's game more often than 3 times out of 4.

While it was still impossible to test this experimentally, the issue quickly turned into an emotional debate. Schrödinger wrote that, if this idea of entanglement was true, he was sorry he ever contributed to it! As far as Bohr was concerned, we need only read his response to the 1935 paper by Einstein,

Podolsky, and Rosen (the EPR paradox[3]) to understand that he took the matter personally and would defend the cause to the last!

Einstein is the greatest among the great scientists for having constructed a local theory of gravitation, centuries after Newton. Before the general theory of relativity in 1915, physics described gravity in a nonlocal way, in the sense that, if one displaced a stone on the Moon, our weight[4] on the Earth would be immediately affected. In principle, one could thus communicate instantaneously across the whole Universe. But with Einstein's theory, gravity became a phenomenon like all the other physical phenomena known in 1915, which propagate at a finite velocity from one point to the next through space. Hence, according to Einstein's relativity, if we shift a stone on the Moon, the Earth and the rest of the Universe are informed of this by a gravitational wave which propagates at the speed of light. It would only be about a second later that an Earthling's weight would be affected, since the Moon is some 380,000 km away.

Just 10 years after his heroic discovery, Einstein, the man who made physics local, found himself once again confronted with nonlocality. Even though quantum nonlocality is very different from the nonlocality of Newtonian gravity, he could not remain unmoved in the face of this threat to his conceptual edifice. It is thus easy to understand his reaction, which was quite logical in the circumstances: why should one place more trust in Heisenberg's uncertainty relations than in determinism and locality?

How Entanglement Enables One to Win Bell's Game

The word 'quantum' used to qualify the new physics of the 1920s arose because the possible energies of an atom are quantised, that is, the energy cannot take just any value, but must select within a certain set of values. In fact there are many physical quantities apart from energy that can only assume a finite number of values, and which are thus said to be quantised. A simple and common situation is that there are only two possible values, a situation which yields a quantum bit, or qubit in physicist's jargon.

The different measurements that can be carried out on a qubit can be represented by a 'direction'. In the case of photon polarisation, this direction is

[3] Rae, A.: *Quantum Physics. Illusion or Reality?*, Cambridge University Press (1986); Ortoli, S., Pharabod, J.P.: *Le cantique des quantiques*, La Découverte (1985); Gilder, L.: *The Age of Entanglement*, Alfred A. Knopf (2008).

[4] As measured on the bathroom scales. But our mass would not be affected, only the attractive force that the Earth and Moon exert on us. Note that the stone should be displaced using a rocket, so as to displace - slightly - the moon's center of mass.

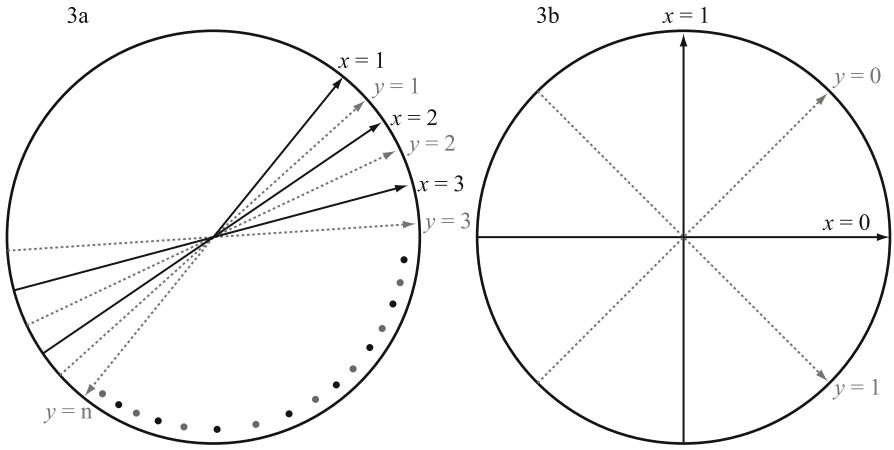

Fig. 5.1 A quantum bit or qubit can be measured in different 'directions'. If two qubits are entangled and we measure them in directions that are close to one another, the results are often the same, hence strongly correlated. For example, in (**a**), the measurement choices $x = 1$ and $y = 1$ give rise to strongly correlated results, as would $y = 1$ and $x = 2$, and $x = 2$ and $y = 2$, and so on. Nevertheless, the choices $x = 1$ and $y = n$ give rise to different results. For Bell's game, Alice and Bob's boxes use the measurement directions indicated in (**b**)

directly related to the orientation of the polariser.[5] We can thus represent these measurement directions as angles on a circle, as shown in Fig. 5.1a. Every time we measure a qubit in one of these directions, we find either the result 0 which corresponds to 'parallel' to this direction, or the result 1 which corresponds to 'antiparallel' to this direction, i.e., pointing in the opposite direction. Reversing the measurement direction simply amounts to swapping 0 and 1, since a result 0 in one direction would be the same as a result 1 in the opposite direction. Note that, for each qubit, we have the choice of measurement direction. Since this measurement perturbs the qubit, we cannot measure the same qubit again in some other direction. On the other hand, we can produce many qubits all of which are prepared in the same way (a physicist would say that they were all in the same state). We can thus choose different measurement directions for different qubits and thereby build up statistics for the given state.

The probability that a qubit produces the result 0 depends on the state in which the qubit has been prepared. But whatever this state may be, the probabilities that the qubit produce the result 0 in two nearby directions are

[5] The polarisation is determined by the direction of the electric field associated with any photon. If the photon is definitely polarised, this vibration will be confined to some precise orientation in space, and this orientation determines the polarisation state of the photon. It is related to possible measurement directions with a factor of 2 in the angles, a factor whose story also deserves to be told.

also close. In other words, there is continuity in the probability of a result as a function of the measurement direction.

If two qubits are entangled[6] and they are both measured in the same direction, we always find the same result, either both 0 or both 1. Why is that? This is precisely the magic of entanglement. As discussed in the section entitled *Quantum Entanglement* on p. 43, each qubit is associated with a cloud of potential results, but the difference between the results of the two entangled qubits is zero. Hence, if Alice and Bob share a pair of entangled qubits and if Alice measures her qubit in some direction A and Bob measures his in some direction B close to A, then the probability of the two results turning out the same is close to 1. Suppose that Bob's direction is slightly to the right of Alice's, as shown in Fig. 5.1a. Now imagine that Alice uses a second direction $\tilde{A}$ that is still close to Bob's, but this time slightly to the right of Bob's. These two directions are once again close to one another so the probability of obtaining the same result for each is once again close to 1.

We can then continue from one point to the next on the circle until the last direction for Bob actually lies opposite to the first direction for Alice. But then for opposite directions, the results will necessarily be opposite! Here we recognise the idea that underpins Bell's game. The results are almost always the same, except in a situation where they are different. In Bell's game, this special case where the results must be opposite to one another corresponds to Alice and Bob both pushing their joysticks to the right. In the scenario with the two entangled qubits, this special case corresponds to Alice using her first measurement direction and Bob using his last. Depending on the number of measurement directions considered, we obtain different Bell inequalities. For Bell's game, Alice and Bob each use only two directions, as shown in Fig. 5.1b. With this strategy, they achieve a score of 3.41.

Quantum Nonlocality

Let us take stock. Quantum theory predicts, and many experiments have confirmed, that nature can produce correlations between two distant events that cannot be explained either by an influence of one event on the other, or by a common local cause. To begin with, we should be a little more precise. What is excluded is an influence propagating continuously from one point of space to the next with any speed up to the speed of light. (In Chaps. 9 and 10,

[6] There are infinitely many possible entangled states. Here I consider the state known to physicists as Φ^+ and measurements in the xz plane.

we shall see that this result can be extended to any finite speed, even speeds greater than the speed of light, provided that they are not infinite.) Similarly, what is ruled out are common causes propagating continuously from point to point through space. These two types of explanation are said to be based on local variables because everything happens locally and evolves from one point to the next, whence the standard terminology 'local explanation' or 'by local variables'.[7]

The really remarkable thing here is that, once we have ruled out any explanation in the form of an influence or a common cause propagating continuously from point to point, there is no other possible local explanation. This means that there is no explanation in the form of a story taking place in space as time goes by and which describes how these famous correlations can be produced. To put it bluntly, these nonlocal correlations seem in some sense to emerge somehow from outside of spacetime!

But are not we jumping to conclusions here? And what are these nonlocal correlations? Let us begin with the latter question, the less difficult of the two. Since these correlations have no local explanation, they are said to be nonlocal. So nonlocal simply means 'not local', or more pedantically, not describable in terms of local variables. The qualifier 'nonlocal' is thus negative: it does not tell us what these correlations are actually like, but rather what they cannot be like. It is as though we were told that an object is not red. That would not tell us what colour the object actually is, only that it is not red.

One very important aspect of the fact that 'nonlocal' is a negative qualifier is that it does not at all mean that nonlocal correlations can be used to communicate, either instantaneously or at a speed greater than or less than the speed of light. In no way do nonlocal quantum correlations provide a means of communication. Nothing we are able to control in experiments with nonlocal correlations goes faster than light. There is no transmission, hence no communication, but the observed results cannot be explained by local models, i.e., they cannot be accounted for by stories taking place in space and time.

The fact that there is no communication avoids a situation where quantum physics would be in direct conflict with relativity theory. Some would describe it as a peaceful coexistence,[8] a somewhat surprising way of talking about the two cornerstones of modern physics. These cornerstones nevertheless rest upon foundations that are totally conflicting. Quantum physics is intrinsically random, while relativity theory is profoundly deterministic. Quantum physics predicts the existence of correlations that simply cannot be described using local variables, while everything in relativity theory is fundamentally local.

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[7] Some prefer to speak of local hidden variables, but whether or not they are hidden changes nothing here.

[8] Shimony, A.: In *Foundations of Quantum Mechanics in the Light of New Technology*, ed. by S. Kamefuchi et al., Physical Society of Japan, Tokyo (1983).

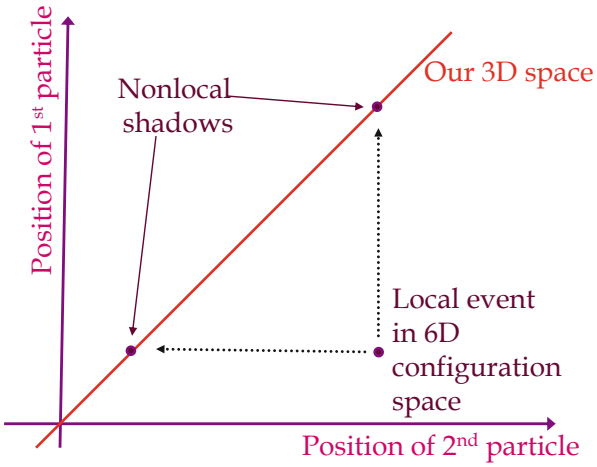

Fig. 5.2 Quantum theory uses a high-dimensional space to describe particles. For two particles only allowed to propagate along a *straight line*, this space is 2-dimensional, like a sheet of paper. Hence each *point* on the figure represents the positions of two particles. The *diagonal* represents our usual space. An event in the large space then projects two shadows in our space and these shadows may be widely separated from one another

Origin of Quantum Correlations

To end this chapter, let us ask how the mathematical formalism of quantum physics describes nonlocal correlations. After all, the formalism works very well. Can it not explain how these nonlocal correlations come into being.

According to this formalism, these peculiar correlations arise from entanglement and this in turn is described through a kind of wave that propagates in a much bigger space than our 3-dimensional space. This space in which such 'waves' propagate, known to physicists as configuration space, has a dimension that depends on the number of entangled particles, in fact, 3 times the number of entangled particles. In configuration space, each point represents the positions of all the particles, even if they are widely separated from one another. In this way, a local event in configuration space can involve quite remote particles. But we simple human beings cannot see configuration space, only shadows of what is actually going on there. Each particle projects a shadow in our 3-dimensional space, a shadow corresponding to its position in our space. The shadows of a point can thus be far removed from one another in our space, even if they are the shadows of a single given point in configuration space (see Fig. 5.2). This is indeed a weird explanation, insofar as it can be called an explanation. In a certain sense then, reality is something that happens in another space than our own, and what we perceive of it are just shadows,

rather as in Plato's cave analogy used centuries ago to explain the difficulty in knowing the 'true reality'.

This 'explanation' for the origin of the nonlocal quantum correlations seems more mathematical than physical. Indeed, it is difficult to believe that the true reality should take place in a space whose dimension depends on the number of particles, especially when we recall that this number changes over time. In short then, the mathematical formalism of quantum theory provides no explanation, only a way of doing calculations. Some physicists conclude that there is nothing to explain: "Shut up and calculate", they would advise us.

6
Experiment

In this chapter, I would like to acquaint you with a Bell experiment, in fact, the one we carried out in 1997 in Geneva, or more precisely between the villages of Bernex and Bellevue, about 10 km apart as the crow flies, using the optical fibre network of our national telecommunications operator Swisscom. Figure 6.1 shows the configuration of this experiment, the first Bell experiment to be done outside the laboratory.

Producing Photon Pairs

Let's begin at the very heart of the experiment with the production of the entangled photons. In a crystal, the atoms are lined up in a highly regular way (but note right away that the crystal used as a source for entanglement has nothing to do with the crystals in Alice and Bob's boxes). Each atom is surrounded by a cloud of electrons. When these atoms are excited by shining light on them, the electron clouds oscillate around the atomic nuclei. If this oscillation is asymmetric, that is, if the electron clouds move away from the nuclei more easily in one direction than in the other, then we have what is known as a nonlinear crystal. The reason for this name is as follows. When a photon interacts with an atom, it excites the electron cloud, causing it to start oscillating. If the cloud oscillates symmetrically, it de-excites by emitting a photon just like the initial photon in a completely arbitrary direction. This is known as fluorescence. On the other hand, if the cloud oscillates asymmetrically, the cloud relaxes by emitting a photon of a different colour.

But the colour of a photon determines its energy, and one of the basic laws of physics is that energy is conserved. The above description is thus necessarily incomplete. However, there are nonlinear crystals which, when illuminated with infrared light, produce a beautiful green light. This is the physical mechanism in the green laser pointers which have been becoming more and more common in conferences over the past few years. The explanation here is that one requires two low energy photons, in the infrared part of the spectrum, to produce a single photon of higher energy, in the green. The

N. Gisin, *Quantum Chance,*
DOI 10.1007/978-3-319-05473-5_6, © Springer International Publishing Switzerland 2014

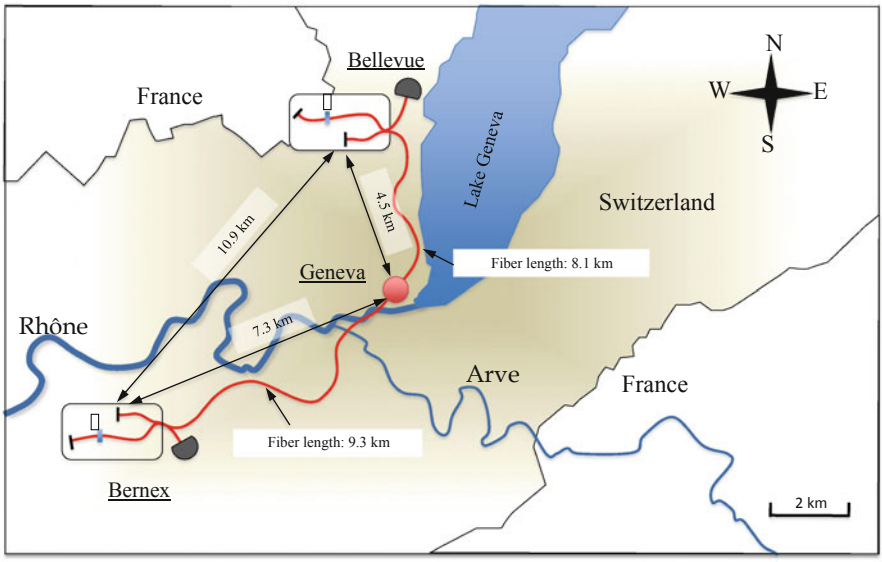

Fig. 6.1 Nonlocal correlation experiment between the villages of Bernex and Bellevue, some 10 km apart as the crow flies. When we carried out this experiment, it was the first time anyone had played Bell's game outside the laboratory. The optical fibres used to propagate the entangled state were those of the commercial telecommunications network Swisscom

intensity of this green light thus goes as the square of the infrared intensity,[1] hence the name 'nonlinear'. A nonlinear crystal can therefore modify the colour of a light beam. On the photon level, this process necessarily involves several low energy photons.

The laws of physics are reversible. This means that, if an elementary process can occur in one direction, the reversed process must also be possible. It must therefore be possible to send a green photon into a nonlinear crystal and generate two infrared photons. So here we have a process that can produce photon pairs.[2]

[1] To produce a green photon, two infrared photons must simultaneously appear, by chance, at the same point in the crystal. The probability of this event varies with the square of the infrared light intensity.

[2] Depending on which nonlinear crystal is used, these two photons will not necessarily have the same average colour. For example, one may be a light-coloured infrared, hence containing a little red, while the other is a dark-coloured infrared and thus totally invisible to our eyes. This difference of colour, and hence also of energy, can be quite significant, and in particular, greater than the indeterminacy in the energy of each of these photons, although we shall continue to refer to them as infrared. Thanks to this difference, the two photons can be separated and the light infrared one sent to Alice, for example, while the dark one is sent to Bob. To do this, the photons are injected into optical fibres, the same kind of fibres as those used every day when you surf on the Web, watch television, or phone a friend. In the actual experiment, the

Producing Entanglement

It remains to see why these photons are entangled. To understand this, remember that quantum particles like photons generally have indeterminate physical quantities (position, velocity, energy, and so on). For example, a photon has some energy, but this energy is indeterminate. On average it may have this or that value, but with an indeterminacy that may be quite large. This is not an uncertainty in our knowledge of the photon energy, but an indeterminacy *intrinsic* to the photon, which does not itself 'know' its energy. Put briefly, the photon does not possess a precise energy, but a whole spectrum of potential energies (just like the position of an electron, as described in Chap. 5). If this energy is measured very accurately, we obtain a random result from the spectrum of possibilities, a result of the truly random kind discussed previously. It should be understood here that, in order to produce true randomness of the kind we have seen to be essential if we are to win Bell's game, certain physical quantities must not have precisely determined values. They must be indeterminate and only assume a precise value when they are measured precisely. What precise value? That's quantum randomness.

Just like its energy, the age of a photon, that is, the time elapsed since it was emitted by a light source, can also be indeterminate. Hence, the set of potential ages of a photon can range over several billionths of a second or up to a few seconds, depending on how the photon was emitted. For photons, the famous Heisenberg uncertainty relations (see Box 8 on p. 38) states that, the better determined the photon age, the greater the indeterminacy in its energy. And conversely, the better determined the photon energy, the greater the indeterminacy in its age.

But let's go back to our nonlinear crystals and the photon pairs they generate. Imagine that a nonlinear crystal is excited by a green photon with very precise energy, that is, for which the indeterminacy in the energy is very small. The photon is transformed into two infrared photons, each of which has an indeterminate energy but in such a way that the sum of the energies of the two infrared photons is exactly equal to the energy of the original green photon. We thus have two infrared photons, each with indeterminate energy, but for which the sum of their two energies is very accurately determined.

The energies of the two photons are thus correlated. If we measure these energies and if we find a value above the average for one of the photons, the other will necessarily have an energy below the average. Here we have a surprising

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infrared photons are adapted to the characteristics of the optical fibres. One speaks of telecom photons. Their colour is such that the transparency of the optical fibres used in telecommunications is maximal.

feature of nonlocality: the energy of a photon, initially indeterminate, can be determined as a result of a measurement carried out on the other photon.

But this is still not quite everything we need. In order to play Bell's game, we must have a choice between at least two types of measurement, corresponding to the two positions of the joystick. Since the original green photon has a very precise energy, in order to respect Heisenberg's uncertainty principle, it must have a correspondingly indeterminate age. So what about the infrared photon pairs? As their energies are indeterminate, their ages can be determined relatively accurately, in fact much more accurately than the age of the green photon.

Could one of the infrared photons be older than the other? The answer is negative, since that could only happen if this photon had been produced by the crystal before the other. If one infrared photon came into existence before the other, there would be a short interval of time during which the law of conservation of energy would not be satisfied, which is impossible. The two infrared photons must therefore be created simultaneously, at the same time as the green photon is destroyed. What is this time when the two infrared photons are created? The answer is that the instant of creation of the two infrared photons is indeterminate, just like the age of the green photon.

To sum up then, the two infrared photons have the same age, but this age is indeterminate. If we measure the age of one infrared photon, we find a truly random result. But from this moment, the age of the second photon becomes determinate. So here is the second quantum correlation we need in order to play–and win–Bell's game.[3]

Once the photon pairs arrive at their destinations, one in Alice's box and the other in Bob's, they should ideally be stored in memory. This kind of memory, called a quantum memory, is still under development in research laboratories. At the present time, they are not very efficient and can only store a photon for a tiny fraction of a second. As a result, Alice and Bob are asked to make their choice slightly before the photons arrive. In this way, they are immediately measured as soon as they arrive in the boxes. Depending on the positions of the joysticks, they will undergo one or other of the two possible measurements, that is, of the energy or the age (energy or time, as physicists

[3] If we accept Heisenberg's uncertainty relations and hence that measurements in quantum physics produce intrinsically random results, then we don't require two physical quantities like the energy and the age of the photons. Just one would be enough to demonstrate the nonlocality of quantum physics. But if there were not two quantities, nobody would believe in true randomness, because they could say, for example, that the energy of each photon was perfectly determinate, but that we just don't know its value. It is only by virtue of Bell's game, in which it is essential that Alice and Bob should be able to choose between (at least) two options, that we can convince ourselves of the existence of true randomness and the validity of Heisenberg's uncertainty relations.

would say). Finally, each box will display the result of these measurements. But in principle, enough photons could be stored in Alice and Bob's boxes to play Bell's game as described in Chap. 2 (and the technology will soon be available to achieve this). The crystals at the heart of the two boxes are thus quantum memories which store several hundred entangled photons, like the quantum memory crystals we are currently developing in Geneva (but with a storage time and efficiency that we shall have to improve significantly).

Entanglement of Quantum Bits

We have just seen how to produce two infrared photons that are entangled in energy and in age. If we measure the energy or the age of these two photons, we obtain perfectly correlated results. The joysticks of Alice and Bob's boxes can determine whether the box measures the energy or the age of the photons, but this is not yet enough to be able to play Bell's game. The reason is that, in this game, the boxes must produce binary results, while energy or age measurements produce numerical results that may assume a wide range of possible values (in principle, infinitely many different values). We must therefore discretise the entanglement, so to speak.

We can start by replacing the laser that shines continuously on the nonlinear crystal by one that produces short pulses of light. Splitting this pulse in two using a semi-transparent mirror, known to physicists as a beam splitter, and delaying one of the two half-pulses, we recombine them as shown in Fig. 6.2. The crystal is then illuminated by a series of two half-pulses. This is still the nonlinear crystal that we use to produce the photon pairs. When will these photon pairs be produced? Each green photon from the laser is split in two, one half being delayed, and the result is sent to the crystal. So each green photon can be transformed into two infrared photons in the nonlinear crystal at two different times. If we detect one of the infrared photons, we may find it at two different times, either on time or delayed. And then the other infrared photon will necessarily be found at the same time, i.e., it will have the same age. So here we have a binary result for the age measurement on the infrared photons. (It is important to understand that the green photon is not sometimes on time and sometimes late, but always on time *and* late, in a superposition of the two, as the physicist would say. Its cloud of potential ages has two peaks, one corresponding to being on time and the other to being late. Each pair of infrared photons produced by a green photon is also both on time and delayed in this sense, but the two photons always have the same age.)

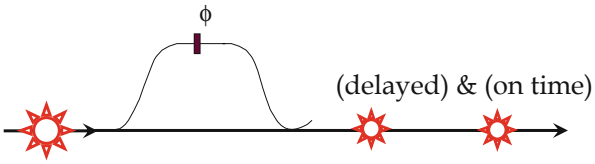

Fig. 6.2 Illustration of a time-bin quantum bit (*qubit*). On the *left* is an incident photon. This photon may follow the short path (at the *bottom* of the figure) or the long one (at the *top*). These two paths are then recombined. The photon can thus be 'on time', if it takes the short path, or 'delayed', if it takes the long one. According to quantum physics, the photon can in fact take both the long path and the short path, whereupon it will be both on time and delayed (or in a superposition, to use the physicist's terminology)

The second measurement required here to play Bell's game, the energy measurement, requires an interferometer. The important thing to understand is that one can also discretise the energy measurement[4] and thus play and win Bell's game.

The Bernex-Bellevue Experiment

We carried out the above experiment in Geneva in 1997. This was the first time that Bell's game had been played outside a physics laboratory. I had a solid background in standard telecommunications, and optical fibres in particular, having contributed to their introduction in Switzerland at the beginning of the 1980's. The main technical problem was to be able to detect one by one

[4] The interferometer is used to delay the part of the photon that is on time and so make it coincide with the delayed part of the same photon. In this way, the two parts of the infrared photon reaching Bob are brought together on a fibre optic coupler, the equivalent of a semi-transparent mirror. The photon has the choice between the two exits from the interferometer, each being equipped with a photon detector. We thus have a binary result once again. Each of the two interferometers is equipped with a phase modulator. In practice, this is a component that serves to slightly lengthen the optical fibre, thereby delaying the part of the infrared photons that is on time. This lengthening is tiny, less than the wavelength of the photons, so it does not alter the fact that the two parts of each photon meet up on the last coupler of each interferometer at the same time. To achieve this, a piezoelectric element can be used to stretch the optical fibre very slightly, for example. The important thing is that the two photons should always have one chance in two of being detected by each of the two detectors. On the other hand, the probability that the two infrared photons of one pair should both be detected by the upper detector, and hence that $a = 0 = b$, depends on the way in which we lengthen the optical paths at Alice's location and at Bob's, or as physicists would say, it depends on the sum of the phases. Hence, the correlation between Alice's and Bob's results depends on these slight extensions with Alice and with Bob. Formally, this form of entanglement, known as time bin entanglement, is equivalent to polarisation entanglement [W. Tittel, G. Weihs: Quantum Information and Computation **1**, 3–56 (2001)]. It has the advantage of being well suited to optical fibres, while it is also easy to increase the number of time bins and hence explore cases with far more than just two possible results.

photons that had wavelengths compatible with the optical fibres. At the time, such detectors did not exist. During our first runs of the experiment, we dipped some diodes in liquid nitrogen to keep them at low temperature. A quite different kind of problem was to have access to the optical fibre network of our national telecommunications operator Swisscom. Fortunately, I had excellent contacts there, thanks to my earlier work in telecommunications.

The crystal used as a source of entanglement and all the accompanying equipment was transported and set up in a telecommunications center near the railway station in Cornavin. From there, one optical fibre leads unbroken to the village of Bellevue to the north of Geneva, while another leads to Bernex, a village to the south of Geneva, more than 10 km from Bellevue as the crow flies. In each village, we were able to set up our interferometers and photon detectors (complete with liquid nitrogen!) in small telecom centers. Naturally, a key was needed to gain access to these centers. After opening the door, one is allowed just one minute in which to contact the alarm center to give the password using a special-purpose interphone. One can then go down to the fourth basement floor where the optical fibres arrive from all over the region. Since it is impossible to use a mobile phone there, the reader is invited to contemplate the logistic problems!

And so began the experiment. We were confidently expecting to win at Bell's game, but three surprises were in store for us. The first was that, when the sun rises, the fibre heading south gets significantly longer than the other. And yet they were very much of the same length. The probable explanation is that this fibre goes over a bridge and, being less deeply buried than the other, undergoes greater temperature variations. That raised a difficult problem of synchronisation, although several sleepless nights brought a solution. The second surprise was a pleasant one. Mrs. Mary Bell, John Bell's widow, came to see how we were getting on. Finally, after publication of the experiment,[5] we were treated to a third surprise in the form of a major article in the New York Times, a visit from the BBC to film what was going on, and a nomination by the American Physical Society as one of the experimental highlights of the 1990s.

[5] Tittel, W. Brendel, J., Zbinden, H., Gisin, N.: *Violation of Bell inequalities by photons more than 10 km apart*, Phys. Rev. Lett. **81**, 3563 (1998).

7
Applications

A relevant physical concept necessarily has consequences in everyday life. The equations of electrodynamics discovered by Maxwell in the nineteenth century were largely responsible for the development of electronics in the twentieth century. Likewise, we should expect quantum physics, discovered in the twentieth century, to drive technological development in the twenty-first century. Quantum physics has already given us lasers, used in DVD readers, for example, and the semiconductors so important in computers. But these first applications exploit only properties of ensembles of quantum particles, that is, ensembles of photons in lasers and ensembles of electrons in semiconductors. So what about applications of nonlocal quantum correlations? These involve pairs of quantum particles, one for Alice and one for Bob. These particles must therefore be handled individually, and that is a tremendous challenge. But physicists are not the kind of people to stand back and watch. This chapter presents two applications that have already been commercialised, but it is highly likely that many other wonderful applications are just waiting around the corner.

Random Number Generation Using True Quantum Randomness

The first application is extremely simple. We have seen that nonlocal correlations are only possible if the results obtained by Alice are truly random. But what can randomness do for us? Well, there is nothing more useful in our information society. We all possess a credit card and countless passwords. Our credit card has a PIN code which must remain secret, that is, chosen at random. But it is not so easy to generate randomness. Earlier we discussed the importance of random numbers for numerical simulation. Another example developing very quickly these days is online Internet gambling. Once again, one must be sure that, when a virtual card or winning number is drawn, it really is the result of chance selection. Otherwise, either the electronic casino is cheating, or, if it is just using pseudo-random numbers, there is a risk of some smart

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individual identifying the sequence and bringing the casino to ruin. Consequently, a very promising application for quantum physics is the development of a random number generator exploiting intrinsic quantum randomness, the only true randomness known to physics.

Applied physics is all about understanding some aspect of physics sufficiently well to be able to simplify a protocol until it can be achieved in an economically viable way. Using two computers, Alice and Bob, separated by some spacelike interval so that they cannot influence one another at the speed of light but winning at Bell's game is much too complicated a scenario to envisage a commercial application. If we consider Alice alone, we see that what she has is basically a stream of photons that go through a semi-transparent mirror before being intercepted by two photon detectors. The fact that there is entanglement and that Bob operates likewise at his end to win Bell's game is enough to ensure that Alice's result is indeed due to true randomness, but at the end of the day, we only need Alice's result here. It thus suffices that Bob be possible in a virtual sense. So for the application, we can forget Bob. And once we have taken this step, entanglement is no longer necessary. It is enough to know that Alice's photon could in principle be entangled, but in practice there is no need for it to actually be entangled. Finally, rather than a single photon, Alice can use a highly attenuated laser source, so that there is almost never more than one photon. This is the basis for most of the commercially available quantum random number generators (QRNG).

Figure 7.1 shows the QRNG commercialised by the Genevan company ID Quantique SA.[1] One might think that it is just too simple. What has happened to the nonlocal correlations? This generator does not use them directly, but it is just the *possibility* of using the same type of photons, beam splitters, and photon detectors to produce nonlocal correlations that ensures that the results really are obtained purely by chance.

Some may be suspicious and ask how we can be sure that we have the same kind of beam splitters and detectors? They are quite right. To simplify this random number generator enough to make it commercially viable, we had to make the hypothesis that the devices are reliable. This assumption is very common and well tested. There is a very elegant way to get round it, but then we would first have to come back to a situation much closer to the one in the Bell game and give up most of the simplifications listed above. This has been done experimentally, but only in the laboratory.[2]

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[1] `www.idquantique.com`.

[2] Pironio, S., et al.: *Random numbers certified by Bell's theorem*, Nature **464**, 1021–1024 (2010); B.G. Christensen, K.T. McCusker, J.B. Altepeter, B. Calkins, T. Gerrits, A.E. Lita, A. Miller, L.K. Shalm, Y.

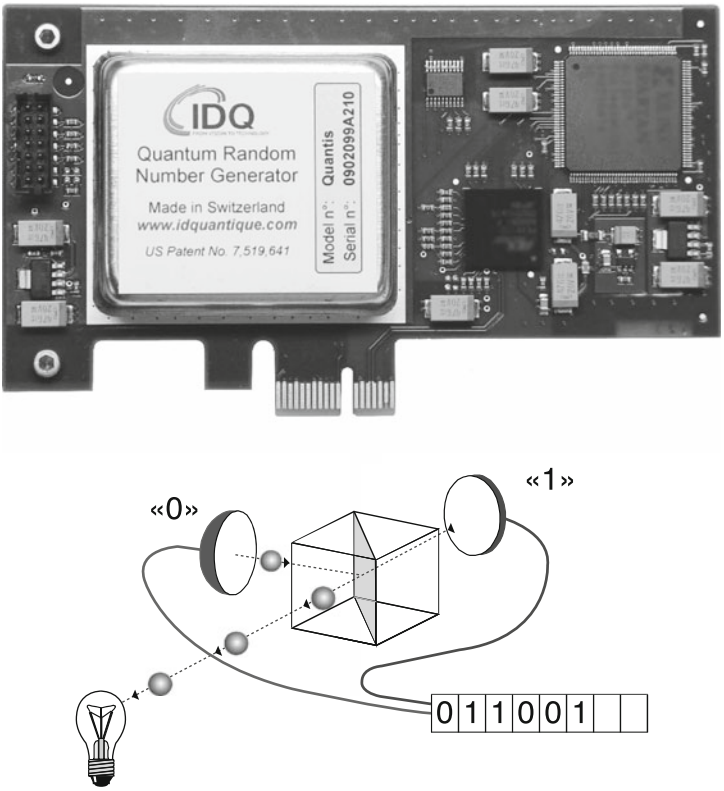

Fig. 7.1 Quantum random number generator. The idea is illustrated in the diagram. A photon passes a semi-transparent mirror to reach one of two detectors. Each detector is associated with a binary number, or bit. At the *top* is the first commercial generator, measuring 3 × 4 cm, as made by the Geneva-based company ID Quantique

Quantum Cryptography: The Idea

A second application is quantum cryptography. We have seen that, if two objects are entangled, they always produce the same result if we carry out the same measurement on each of them. At first glance, that does not look particularly useful, especially since these identical results are produced by pure chance. But for a cryptographer, all that looks extremely interesting. Indeed, our information society channels around enormous amounts of information, a large part of which needs to remain confidential. To achieve this, the information is encoded before being sent to its recipient. What that means is that, to the eyes

Zhang, S.W. Nam, N. Brunner, C.C.W. Lim, N. Gisin, P.G. Kwiat: *Detection-loophole-free test of quantum nonlocality, and applications*, Phys. Rev. Lett. **111**, 130406 (2013).

of any third person, the coded information just looks like a long string of noise without structure or meaning. But in the long term, it is essential to change the code at regular intervals, ideally for each new message, and that raises the question of how to exchange coding keys. These keys must be known to both the sender and the receiver, but to nobody else. We might imagine fleets of armoured taxis driving around the world to distribute these keys to their users, but surely there is a simpler way?

Today, some governments and large companies do in fact send physical persons with an attaché case fixed to their wrist to distribute coding keys to partners with whom they consider it absolutely essential to communicate in a highly confidential way. Ordinary mortals like ourselves are quite happy with a more practical system, e.g., for Internet shopping, in which security is based on the mathematical theory of complexity. This uses what is known as public key cryptography. The idea is that certain mathematical operations such as multiplying together two prime numbers are easy to carry out using a computer but very difficult to invert. In this example, one would have to find the two prime numbers from the value of their product, a potentially long task, even for a powerful computer.

But the details are not important here. What counts is to understand what difficult means here. For a schoolchild, a problem is difficult if even the best pupils are unable to crack it. In public key cryptography, it's exactly the same thing, except that instead of one's classmates, one takes the best mathematicians in the world, brings them all together in a suitably comfortable place, and promises them a lavish reward if they should succeed in finding the solution. If none of them find it, that means the problem was genuinely difficult. But then difficult does not mean impossible. The history of mathematics is full of examples of problems that have stumped the world's best mathematicians for years, sometimes even centuries, before some mastermind has come up with the solution.

Mathematics is such that, as soon as we have a solution, it is never difficult to reproduce and exploit. So if one day, tomorrow for example, some gifted mind should discover a quick way to find the two prime factors hidden behind their product, all the electronic money in our society would instantaneously lose all value. There would be no more credit cards, no online business, and no interbank loans. That would be a major disaster. In addition, if an organisation had recorded communications encrypted by public key, it could subsequently decipher them and read confidential messages sent years or even decades earlier. So if you really want your data to remain confidential for decades to come, you had better give up public key encryption right away.

Hence the importance of finding results that occur by pure chance but are always identical for Alice and Bob. If Alice and Bob share entanglement, they

can at any moment produce a sequence of results that they can use right away as an encoding key. And thanks to the no-cloning theorem, they can be sure that nobody else will ever come to have a copy of their key. It's as simple as that, at least on paper.

Quantum Cryptography in Practice

How can we simplify the setup of the Bell game for a practical application of the above idea? Once again, we shall see how important it is to understand the underlying physical principles in order to develop a simple, but not oversimplified, implementation of quantum cryptography.

First Simplification There are three parts to each experimental run of Bell's game: Alice, Bob, and the crystal producing the entangled photons. For reasons of symmetry, the latter is usually located in the middle. However, that is not very convenient, so let us store it with Alice. In that way we only have two entities, but in doing so, we no longer have the block on communication between Alice and Bob imposed by relativity. However, in cryptography, one must already ensure that no information leaks out against their will, since that would already constitute a breach of confidentiality.

Second Simplification Now that the source of entangled photon pairs is with Alice, she measures the qubit carried by her photon well before Bob. In fact, she measures it even before the other photon has left Alice on its way to Bob. So rather than using a source of photon pairs and measuring one of them (and hence destroying it) immediately, it is much simpler for Alice to use a source that just produces photons one by one.

Third Simplification A source producing photons one by one is still complicated. It is even simpler to use a source that produces extremely weak laser pulses, so weak that one pulse rarely contains several photons. And that is indeed a reliable, well-tested, and cheap source. The only remaining problem is to decide what to do with the rare cases of multiphoton pulses. In practice, one only needs an accurate estimate of the frequency of these multiphoton pulses. Then one makes the conservative assumption that any spy would know everything about these multiphoton pulses. After exchanging many pulses, usually millions of them, Alice and Bob can estimate how much their enemy knows about their results in the worst possible scenario. They can then use a

standard privacy amplification algorithm.[3] This allows one to extract a slightly shorter key from a longer one with the guarantee that an adversary holds at most an exceedingly small amount of information. Although the new key is shorter, one can be certain that it is absolutely secure.[4]

When all is said and done, there are only two boxes. One sends very low intensity laser pulses which carry polarisation-encoded or time-bin encoded quantum information, as described in Chap. 6, while the other measures the polarisations or the ages of these photons. In practice, there are of course other technological tricks, but if you have been able to follow up to now, you will have understood a large part of applied physics.[5]

Today, some organisations in Geneva with their data backup systems 70 km away near Lausanne use optical fibres passing under Lake Geneva to avail themselves of cryptography systems commercialised by the Genevan company IDQ, a spinoff of the University of Geneva,

Historically, it is interesting to observe that the above simplified version was invented well before the one based on nonlocality. Here is one of the anomalies of history, so very human, which does not always follow the path of logic. For another very human story, when Bennett and Brassard invented quantum cryptography in its simplified version, no physics journal would publish it. Too novel! Too original! Hence incomprehensible to those physicists invited to assess their work for publication. In the end, Bennett and Brassard published their result in the proceedings of a computing conference in India. Needless to say, this 1984 publication went completely unnoticed until the independent rediscovery of quantum cryptography by Artur Ekert in 1991, a discovery this time based on nonlocality and published in a prestigious physics journal.

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[3] Intuitively, this works as follows. Imagine 2 bits b_1 and b_2 and an adversary who has a probability of 3/4 of guessing each of these bits correctly. Replace these two bits by their sum $b : b_1 + b_2$ (modulo 2, so that their sum is still a bit). The adversary correctly guesses b only if he guesses both bits correctly or if he guesses both incorrectly. This means that he guesses b correctly with probability

$$\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{5}{8},$$

which is less than 3/4. So Alice and Bob have thereby increased the confidentiality of their key at the cost of losing half of it. More sophisticated algorithms can do better while losing significantly less of the original key.

[4] The initial insecurity must not be too great. For this reason, the pulses that Alice sends to Bob must be weak enough to limit the frequency of multiphoton pulses.

[5] For more details, see N. Gisin, G. Ribordy, W. Tittel, H. Zbinden: *Quantum cryptography*, Rev. Mod. Phys. **74**, 145–195 (2002); V. Scarani, H. Bechmann-Pasquinucci, N. Cerf, M. Dusek, N. Lukenhaus, M. Peev: *The security of practical quantum key distribution*, Rev. Mod. Phys. **81**, 1301 (2009).

8
Quantum Teleportation

What could be more amazing than teleportation? An object disappears here, only to reappear over there without ever passing through any intermediate points! The communication technologies have sometimes given consideration to this. An email leaves my computer and a few seconds later appears on the computer screen of a friend on the other side of the world. But for an email, we know very well that a whole network of Wi-Fi signals, electrons in copper cables, and photons in optical fibres must have carried my email continuously from one point to the next through space until it reached its destination. Teleportation is quite different, because the object 'jumps' directly from here to there without anything passing through any intermediate point. It smacks of magic, or science fiction. Unless there is some way of exploiting quantum nonlocality, this uncanny link between distant places.

Throughout this book, we have seen that nonlocality cannot be used to communicate. But the science fiction version of teleportation does allow communication at arbitrary speed. Moreover, any object is necessarily made up of matter (or energy if it is a photon) and matter cannot go from one place to another without passing through some intermediate points. The science fiction variety of teleportation is therefore impossible. And yet in 1993 a group of physicists having fun in a brainstorming session began to play around with the idea of nonlocality and invented what we know today as quantum teleportation.[1] The publication has six authors, so nobody invented quantum teleportation on their own. It really was the result of intellectual ping-pong

[1] Here is a little story to illustrate the ground that has been covered since the beginning of the second quantum revolution in the 1990s. In 1983, when I was a young Postdoc in the United States, an important professor came up to me with a broad smile on his face and informed me that he had saved my life. He admitted having been the referee for one of my first scientific publications, in which I had committed the unforgivable blasphemy of writing that, in quantum physics, it seemed possible "that a system might disappear here and reappear somewhere else". Today that is reminiscent of teleportation, but in reality I was not thinking of that at all. It was just an intuition. My 'saviour' had accepted my paper for publication solely on the condition that the above-mentioned blasphemy be removed. At the time, my assertion would have earned me universal disapproval! One may wonder how many opportunities have been lost thanks to worthy professors endlessly insisting that Bohr had sorted everything out. How many talented young minds may have left physics as a result? And how many great professors still insist that Bohr really did sort everything out?

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within a group of people, far removed from the typical image of the brilliant and isolated savant.[2]

Substance and Form

So how does teleportation get off the ground? To begin with, we must ask ourselves what is meant by an object. Aristotle suggested two essential ingredients: substance and form.[3] Today, physicists would speak of matter and physical state. For example, a letter is made of paper and ink on the one hand, these being matter, and a text on the other, this being the information or the physical state of the paper and ink. For an electron, the substance is its mass and electric charge (along with other permanent attributes), while its clouds of potential positions and velocities constitute its physical state. For a photon, a massless light particle, the substance is its energy and its physical state comprises its polarisation and its clouds of potential positions and vibration frequencies (i.e., energies).

In quantum teleportation, we do not teleport the whole object, only its quantum state, or in Aristotle's terms, its form. Is that disappointing? Not really! To begin with, we know that we could never teleport the mass or energy of an object. That would seriously violate the principle that it is impossible to communicate without transmission (see Box 5 on p. 22). And furthermore, the fact that we are able to teleport the quantum state of an object is quite extraordinary enough. Indeed, the quantum state constitutes the ultimate structure of matter. We are not therefore merely teleporting some kind of approximate description, but *everything* that can actually be teleported. And don't forget the no-cloning theorem of Chap. 4. When we teleport the quantum state of an object, the original must necessarily disappear, otherwise we would end up with two copies, and that would contradict the no-cloning theorem. We are thus guaranteed the disappearance of the original here and the reappearance of the teleported state over there.

So let us sum up. In quantum teleportation, the substance (mass, energy) of the initial object remains at the point of departure, let us say with Alice,

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[2] Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A. and Woottters, W.K.: *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels*, Phys. Rev. Lett. **70**, 1895–1899 (1993).

[3] This was how we began our proposed publication on the first long range teleportation experiment. However, the editors of the famous journal *Nature* rejected any citation going back as far as Aristotle! I strongly urged my students to give up the idea of publishing in *Nature*, but the pressure was too great and in the end we gave in to the editors' demands. I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, N. Gisin: *Long-distance teleportation of qubits at telecommunication wavelengths. I*, Nature **421**, 509–513 (2003) (submitted paper `arXiv:quant-ph/0301178`).

but all its structure (its physical state) simply evaporates. For example, if Alice teleports a plasticine duck, the plasticine will stay where it was but it will no longer have the duck shape. In fact, it will just be a shapeless pile of plasticine. At the destination, let us say with Bob, some arbitrary distance away (and indeed at some location that may remain unknown to Alice), we start out with a shapeless pile of plasticine (the substance). But at the end of the teleportation process, Bob's plasticine will acquire the exact shape of the original duck, right down to the slightest atomic detail. Naturally, this particular example is still a matter of science fiction. For the time being we are unable to teleport a plasticine duck. That would be much too complex for present day technology. Perhaps quantum physics does not even apply on the scale of such everyday objects? So let us consider a second example that is both more realistic and more abstract: the polarisation of a photon.

A photon is a tiny little package of light energy (physicists speak of electromagnetic energy). Among other things, this energy comprises a weak vibrating electric field. If the photon has a well-structured polarisation, the electric field vibrates regularly in a precise direction. However, if the same photon has a structureless polarisation, physicists would say that it was unpolarised,[4] this electric field will vibrate in every direction in a completely disordered way.

Alice's photon starts out with a well-structured polarisation, that is, it vibrates in a well-defined direction. This direction may not be known, but it exists. After the teleportation process, the energy of Alice's photon is still there, but it is no longer polarised. At Bob's end, we start out with a photon (and hence energy[5]) that is not polarised, but after the teleportation process, it has acquired the well-structured polarisation of the teleported photon. Bob's photon is thereafter identical in every way with Alice's original photon, and Alice's photon is in every way identical with Bob's original photon.[6]

So we really do have teleportation here. The photon considered as 'energy + polarisation', or more generally, an object considered as 'substance + physical state', does indeed go from Alice to Bob without passing

[4] For a photon with well-defined polarisation, there is a polariser through which the photon is sure to pass. On the other hand, a completely unpolarised photon always has a one in two chance of passing through any polariser, whatever the orientation of this polariser. In the first case, the photon carries a structure that a polariser can confirm, while in the second, the response 'goes through or doesn't go through' is always 50–50, whatever the polariser, whence the photon has no such structure.

[5] As we saw in Chap. 6, the energy of a photon can be indeterminate. In fact, the same goes for the mass, e.g., of a Bose–Einstein condensate. The important thing is that the substance, mass or energy, should already be present, at least potentially, at the destination.

[6] For physicist readers, this is true if we teleport all the photon's characteristics. If we only teleport its polarisation, the photons will only be indistinguishable if their other characteristics, such as their spectra, are already indistinguishable at the outset.

through any intermediate points of space. After the quantum teleportation process, nothing distinguishes the final situation from a scenario in which one transports Alice's photon to Bob and Bob's photon to Alice.

But none of this tells us how quantum teleportation actually works. We understand that we must make use of quantum nonlocality, but that is not enough. We still require another concept known as joint measurement.

Joint Measurement

To actually achieve teleportation, we thus require an entangled pair of quantum objects. To be more concrete, imagine a pair of polarisation-entangled photons. Next we require an object to be teleported, let us say a photon whose polarisation we would like to teleport. The polarisation state is thus the bit of quantum information or qubit to be teleported. Alice, the emitter, has the photon to be teleported, or more precisely, she has the photon carrying the polarisation qubit to be teleported, and she also has a photon that she knows to be entangled with a third photon in Bob's possession at some great distance away. Alice does not need to know Bob's whereabouts. What can she do? If she measures the qubit to be teleported, she will perturb it and so will no longer be able to teleport the original. If she measures the photon entangled with Bob's photon, she knows that she can produce a nonlocal correlation with Bob, but what could she do with that? All she knows is that, if Bob carries out the same measurement as her, they will both obtain the same result, a random result, but the same at both ends.

The core of the teleportation process requires Alice to exploit a second aspect of entanglement, one that is still poorly understood. Up to now, we have only discussed the first aspect of entanglement, the one that allows two remote quantum entities like two photons to be described by an entangled state. But here, Alice has two photons described by two states. The first is in a well-defined polarisation state, but which may remain unknown to Alice, and the second is in an entangled state. What Alice must do is to entangle her two photons. To do this, it is not sufficient to measure just one or the other. Rather she must measure them jointly. This is hard to understand because, like entanglement itself, it is something that is quite impossible to achieve in the everyday world we are familiar with.

To understand this, imagine that Alice asks her two photons the following question: Are you alike? What she is asking then is this: If I were to carry out the same measurement on each of you, would you both give the same answer? In the world of everyday objects, the only way to answer this strange question

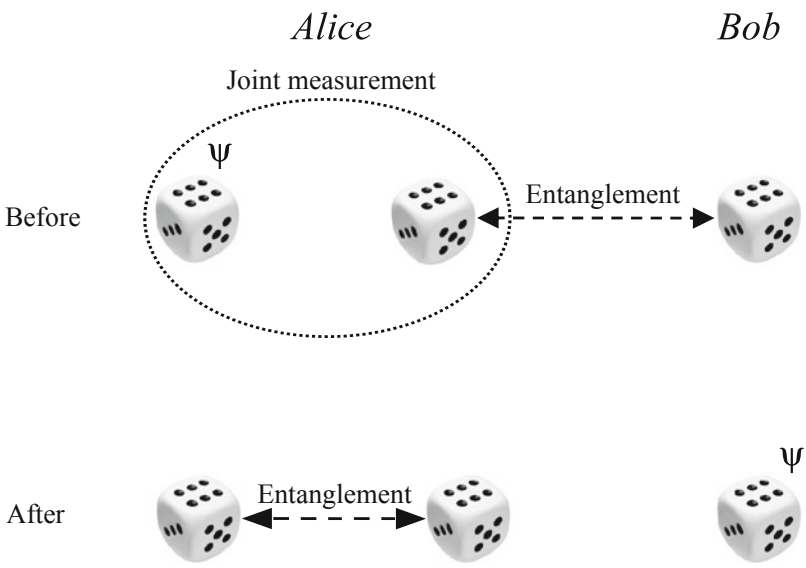

Fig. 8.1 Quantum teleportation. Initially, Alice has two photons represented here by two *dice*. The one on the *left* carries the quantum bit or qubit Ψ to be teleported, while the one on the *right* is entangled with Bob's photon. Alice carries out a joint measurement on her two photons. This operation entangles her two photons and simultaneously teleports the qubit on the *left* to Bob's photon. To complete the process, Alice tells Bob the result of her joint measurement and Bob 'rotates' his photon to accord with this result

is to actually carry out the two measurements and then compare the results. But we can do better in quantum physics thanks to entanglement. We can 'ask' the two photons this question and they will respond by getting together in an entangled state, and this without needing to carry out two measurements, one on each. We already know that an entangled state is such that, if we measure the two photons in the same way, i.e., in the same direction, as discussed in Chap. 5, they will always produce the same random result, characterised as always by true nonlocal randomness. And this will work whatever measurement direction is selected!

If Alice's two photons always produce the same response for the same question, and if Bob's photon, the one that is entangled with Alice's, also produces the same result for the same question, then Bob's photon will always produce the same response as the photon they wish to teleport. It's as simple as that, or almost. We must therefore use entanglement twice, once as a nonlocal quantum teleportation channel (the entangled state of Alice and Bob's photons) and a second time to provide a way of asking two systems (Alice's two photons) a question about their relative state, without obtaining any information about the state of either of them (see Fig. 8.1).

But this is not quite the end of the story. As always in quantum physics, the joint measurement on Alice's two photons, concerning their relative state, produces a truly random result, just one among several possibilities. If we are lucky and we obtain the result 'we are alike', this looks like the end. Except that Bob does not know this. And what would happen if Alice obtained the result 'we are not alike', that is, 'in answer to the same question, we would give opposite answers'? In this case, Bob would have to reverse his photon so that it would be in a state that is also ready to produce the same result as Alice's original photon.[7]

So what does Alice have to do to question her two photons? This is the main experimental difficulty. I won't say more about that here. It would go well beyond the scope of this book.

Quantum Teleportation Protocol

Alice's joint measurement thus produces a random result. Depending on this result, Bob's photon will always produce the result that the initial photon would have produced if it had been measured in the same direction, or else it will produce the opposite result to the one that would have been produced by the initial photon. And these two situations are equiprobable. Up to now, it really isn't very interesting for Bob. He has one chance in two of obtaining the same result as the original photon would have produced, and one chance in two of obtaining the opposite result. To get that far, Bob did not have to do anything at all. Since there are only two possible results, he already knew that he had one chance in two of finding the right result. But in quantum teleportation, Alice knows the results of her joint measurement, so she knows whether Bob finds the right result or the opposite one. To complete the process of quantum teleportation, Alice must therefore tell Bob which situation he happens to be in.

We can now understand how it is that quantum teleportation manages to avoid signalling at arbitrary speed: the process is only complete when Bob has received the result of the joint measurement that entangles Alice's two photons.

[7] Here I must admit to the reader that there are many entangled states. Up to now, to simplify, I have always spoken of entanglement in which the same result is invariably produced for the same measurements. However, there are other entangled states. For example, there are some in which different results are always produced for the same measurements. And in fact, there are many others too, but we shall have no need for them in the present account. For physicist readers, there are four orthogonal states of maximal entanglement for the polarisation of two photons. For each of these, Bob can apply a rotation (a unitary transformation) to the polarisation of his photon in such a way that it ends up in precisely the initial state of Alice's photon, but still without actually knowing what that state was.

This communication between Alice and Bob is needed because, without it, Bob's results are purely chance-like, and Bob has no way of interpreting them. This communication of Alice's result necessarily propagates at the speed of light, or more slowly. Consequently, quantum teleportation, taken from beginning to end, does not go faster than light. Something does indeed happen with Bob when Alice makes her joint measurement, since Bob's photon goes from a state without structure to one of two possible states. Bob has no way of knowing this because, whatever measurement he may carry out, he will obtain a purely random result, but as soon as Alice informs him which of the two states his photon is in, Bob knows what to do in order to systematically obtain the result that Alice would have obtained if she had measured her initial photon–and this whatever measurement Bob may choose to make. Bob's photon is thus in the quantum state of the initial photon.

Note that Bob does not have to measure his photon. He can keep it as it is for future use, or even teleport it somewhere else. We can thus imagine a whole network of teleportation from one node to the next, let us say by 50 km at a time, a distance over which entanglement is easily propagated by optical fibre. If Bob learns from Alice that his photon would always produce the opposite result, he only has to reverse his photon.[8] This can be done without perturbing the photon (Bob reverses his photon without finding out anything about its state). Note also that Bob can teleport his photon somewhere else without rectifying it. He just has to inform its recipient that he must himself carry out the rectification. The final recipient will then calculate how often he must reverse his photon. If it is an even number of times, he need do nothing, and if it is an odd number, he just has to reverse it once.

There is one more important point. Neither Alice nor Bob learn anything about the teleported state. Indeed, the result of Alice's joint measurement on the two photons is always totally random. This result thus provides no information whatever about the teleported state. We should not be surprised about this. We have seen that, starting with an entangled state, the result of a measurement in any specific direction is always totally random, in the sense of being irreducibly chance-like. Conversely, if we start out with a photon that vibrates in a well-specified direction, whatever this direction may be, and we ask it 'Are you alike?', the result is also completely random in that sense. This is a kind of reverse process. And pursuing the argument further, this is absolutely necessary, otherwise if Alice and/or Bob were to learn something about the teleported state, they would be able to repeat the process by teleporting this

[8] To do this, he must rotate the photon state. For example, if the qubit is polarisation encoded, he must reverse the polarisation state using a birefringent crystal.

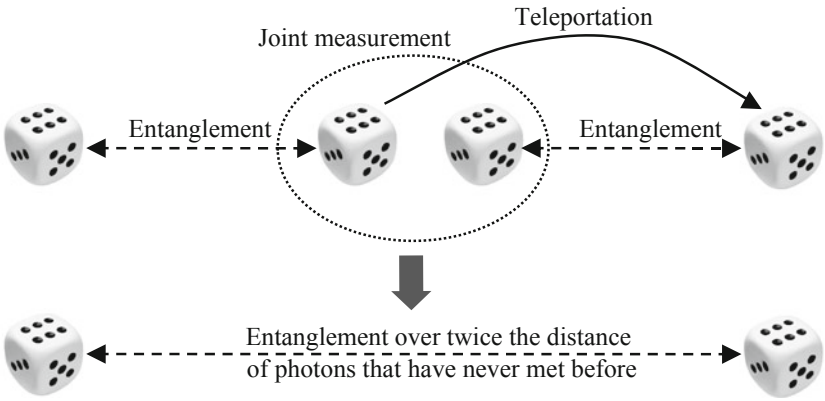

Fig. 8.2 When we teleport a qubit (photon) which is itself entangled, like the *second* on the *left* here which is entangled with the *first*, the result is that the *first* is entangled with the *fourth*. We then speak of teleporting entanglement. This process is particularly fascinating because it entangles particles that have never met before. It is also useful, because it can be used to double the distance between entangled objects

state back and forth from one to the other, each time using a new pair of entangled photons until they had accumulated enough information to be able to produce copies of this state, and that would contradict the no-cloning theorem of Chap. 4.

Finally, Alice and Bob can also teleport the state of a photon that is itself entangled with a fourth photon. Since they learn nothing about the teleported state, they do not destroy the teleported entanglement. We thus exploit both aspects of entanglement, twice to correlate widely separated photons and once to carry out a joint measurement. We may thus entangle photons that have never met and which share no common past, as illustrated in Fig. 8.2. We then speak of teleportation of entanglement.

Quantum Fax and Quantum Communication Networks

It might seem when all is said and done that quantum teleportation is nothing other than a quantum fax system. After all, Bob must already have a qubit corresponding to the blank sheet of paper on which the state of the 'faxed' qubit will be printed. But this analogy is misleading for several reasons.

To begin with, when we use teleportation, we are not just faxing some little piece of information. It is the ultimate structure of matter that gets teleported

in the form of the quantum state. Not only does the final qubit carry the state of the initial qubit, but it is absolutely identical to it in every way.

Secondly, an infinite amount of information is required to describe the state of a quantum system, for there are infinitely many quantum states. For example, the polarisation state of a photon can be described by an angle. To communicate this angle, an infinite number of bits of information are needed, whereas in quantum teleportation, the polarisation state of a photon requires only a single bit. This means that the communication required in quantum teleportation is infinitesimal compared with the amount of information required to communicate the teleported state (if the latter were known).

The third difference is that, in quantum teleportation, neither Alice nor Bob ever learn anything about the state of the teleported qubit. This is quite remarkable and exceedingly useful in cryptography. If someone sends a fax, anyone posted along the line of communication can intercept it. But no such thing can happen with quantum teleportation. As we have seen, nobody learns anything about the state of the teleported qubit, not even the emitter and receiver. So Alice can teleport a message to Charles who subsequently teleports it to Bob. If Charles carries out the quantum teleportation protocol correctly, he learns nothing of the message. Alice and Bob can even check that the process is working correctly and that Charles has found out nothing by applying the quantum cryptography protocol. Generalising to a whole quantum teleportation network, Alice and Bob can ensure confidentiality of their communication even if there are intervening nodes (quantum repeaters, as the physicist would say).

Can We Teleport Large Objects?

Would you be ready to step into a quantum teleportation machine? If I were you, I would be extremely cautious, and this for two reasons. To begin with, the few quantum teleportation experiments carried out so far have achieved proof of principle–and it is wonderful!–but to do so, they have had to select the rare cases where the initial object has not been lost. Indeed, most of these experimental demonstrations use photons, and just as for the demonstrations of Bell's game (the section entitled *Detection Loopholes* on p. 79), many of these photons are simply lost. Physicists know very well why this is and nevertheless consider such demonstrations to be conclusive. However, if I had the choice, I would not readily volunteer myself to take the place of one of these photons in a teleportation experiment. More seriously, several demonstrations have also been made with atoms and in this case almost none of the teleportation candidates are lost. However, the distances involved are so far less than one millimeter.

There is a second reason for being cautious. To teleport an object of everyday dimensions one would require an enormous amount of entanglement. But entanglement is extremely fragile. In order to maintain it, any source of perturbation must be avoided, and this includes any kind of interaction with the environment. This we can do very well with photons which are isolated in optical fibres, or with atoms in special high vacuum traps. But for the enormous amount of entanglement that would be needed to teleport even the tip of a pencil, it would be quite unthinkable at the present time to try to avoid a perturbation that would make the whole teleportation process completely unfeasible.

Today, even with an unlimited budget, nobody has any idea how one could overcome this difficulty. So this is not a simple technical problem. Will we perhaps succeed one day in teleporting the quantum state of a virus? In fact, we are a long way from that today. To begin with, we would need to know what the quantum state of a virus might be. We may even discover that such a thing would be impossible. We may unearth some new principle of physics which forbids the teleportation of an object on our length scale. I have no idea. This is the uncertainty and the beauty of science!

9
Is Nature Really Nonlocal?

To judge by what we have seen so far, it does indeed seem that nature can produce nonlocal correlations. But scientists won't usually let go of a theory or a concept as easily as that. Whenever an experiment delivers strange results, they question not only the theory, but also the experiment. Is it reproducible? Has it been interpreted correctly? In our case, the experiment has been repeated many times and on every continent with all kinds of variants. However, we shall nevertheless see that it is extremely difficult to be sure that all possible alternative interpretations have indeed been ruled out, even though the scientific community is today quite convinced that nature is effectively nonlocal.

In this chapter, we shall review all the different arguments scientists have examined to convince themselves that we must abandon any hope of describing nature in terms of well localised and mutually independent 'elements of reality'. Any picture of nature as being built up in the same way as a child's world might be made from Lego is in fact incompatible with the kind of nonlocality revealed to us by playing Bell's game. Readers who are already convinced and do not wish to work their way through the scientific debate presented below are invited to move directly to Chap. 10.

Nonlocality in the World of Newton

Let us begin with another example of nonlocality. As we have already seen, this is not the first time in history that physicists have encountered nonlocality. The great Newton's theory of universal gravitation was also nonlocal. According to that theory, if we shift a stone on the Moon, our weight on Earth immediately suffers the consequences. This effect, which is immediate whatever the distances involved, is clearly a nonlocal action. But unlike quantum nonlocality, this nonlocal action can be used to communicate without transmission at arbitrarily high speeds. One may well ask how physicists came to accept such a theory for centuries. And the truth is that they never really did accept it. Newton's own reaction says everything (see Box 1 on p. 2): "That Gravity should [. . .] act [. . .] at a Distance [. . .] is to me so great an Absurdity, that

N. Gisin, *Quantum Chance,*
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I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it."

It was only when Laplace came on the scene a few decades later that some thinkers raised Newton's theory to the status of an ultimate truth, deducing from it an absolute form of determinism, and in fact, identifying science and determinism. Newton's own attitude contrasts starkly with that of Niels Bohr, the spiritual father of quantum mechanics, who brought to heel a whole generation of physicists with his insistent assertion that quantum theory is a complete theory. This led Bohr to vilipend the arguments very quickly put forward by Einstein to show that quantum theory is in fact nonlocal. And who knows? This may have prevented some young physicist from finding Bell's argument back in the 1930s. But let us leave these speculations and return to the business at hand.

Today, Newton's nonlocality has disappeared from the theories of physics. Einstein's general theory of relativity has superseded Newton's, and the latter now has the status of an extremely good approximation. According to current theory, displacing a stone on the Moon would only affect our weight here on Earth a second or so later, the time required for a signal to propagate at the speed of light from the Moon to the Earth.

The story of Newtonian nonlocality is relevant here for two quite different reasons. On the one hand one might wonder whether quantum nonlocality might not just be temporary in the same way, to be superseded later on by some theory showing that these correlations which allow us to win Bell's game can be explained locally in space and time and that quantum theory is just a good approximation. But the answer to that is negative. As we have seen, Bell's reasoning is independent of quantum theory, and this argument provides a way of testing nonlocality directly. If we win Bell's game, then nature cannot be wholly described by a local theory, whatever theory it may be.

On the other hand, it is fascinating to note that physics has almost always provided us with a nonlocal description of nature. We had Newton's nonlocality up until 1915, and we have had quantum nonlocality since 1927. So apart from a narrow window of just twelve years, physics has always been nonlocal. We may wonder why even today so many physicists are reluctant to accept nonlocality. But it is no surprise that Einstein should have been among its most ardent critics. After all, he was the one who, after several centuries, finally supplied the answer to Newton by making physics local. The fact that twelve years later another theory would allow nonlocality back into the very heart of physics must have seemed quite intolerable to him. What a pity that nobody during the 1930 s or 1940 s had Bell's brilliant idea. It would have been fascinating to see Einstein's reaction!

Detection Loophole

In Bell's game, every time the joystick is pushed to the left or the right, the boxes supply a result. But in actual experiments of this kind, the photon is sometimes lost[1] or not detected, in which case no result is recorded. Physicists understand very well why certain photons get lost and why photon detectors have limited efficiency. But there nevertheless remains a difference here between the theoretical game and the actual real-life experiment.

In practice, physicists only look at those cases where Alice and Bob's boxes have both produced a result. In other words, they simply ignore the other cases. They assume that the sampling obtained in this way is representative of the whole ensemble and justify this assumption by asserting that nature does not cheat, that is, that nature would not present them with a biased sample. This is cogent reasoning, but since there is nevertheless an assumption here, we must consider that it may conceal some kind of escape from nonlocality.

Imagine that Alice and Bob's boxes use the following strategy. At 9.00 o'clock, each of them produces a result only if the joystick is pushed toward the left (input 0) and also that in this case both results are 0. If the joystick of one of the boxes is pushed to the right, that box simply produces no result. The following minute, they produce a result only if the joystick is pushed to the right (input 1), in which case the responses will be 1 for Alice and 0 for Bob. They then continue in this way, so that each minute each of the two boxes accepts only one question and in that case delivers a predetermined result. If the two boxes have a prior agreement and if we consider only the cases where, by chance, both boxes have produced a result, then Bell's game can be won 4 times out of 4! Indeed, everything happens as though the boxes knew the questions in advance, since they respond only to those questions they are ready (or programmed) to answer. Since there are two possible questions, the probability of getting the right question purely by chance is 50 % for each of the boxes. Therefore, if in a given experiment half of the photons on each side are lost or not detected, it is easy to imagine a strategy for 'winning' Bell's game much more often than 3 times out of 4. One can even 'win' with absolute certainty. I have put scare quotes on 'win' because we are of course cheating here. The boxes do not always answer.

Is it possible that extra local variables might program the photons in such a way that they refuse to answer certain questions, i.e., in such a way that the

[1] In principle, the fact that a photon is lost is not particularly serious, provided we know that before asking it any questions, i.e., before pushing the joystick to the left or right. Otherwise, the photon might decide to lose itself somehow whenever it finds the question unsuitable.

detectors cannot detect them? Most physicists are highly skeptical of such a hypothesis. They feel that they have a very good understanding of the way photon detectors work. In addition, experiments have been carried out with many different types of detector: semiconducting, thermal, and so on. But if we take the extra variable hypothesis seriously, there is really no reason to think that these variables would have no effect on the detection probabilities. Once again, the only good answer is experiment. But no experiment can achieve a detection efficiency of 100 %. One strategy for getting around this difficulty is to decide that, if the physical apparatus produces no answer, we then count these events as response 0. In this way, we always have an answer, but clearly with a majority of zeros.

It can be shown that, using this strategy, one only needs to detect 82.8 % of the photons in Bell's game to exclude any explanation based on extra local variables (see Box 10). However, 82.8 % is still far too high for today's photon technology. Fortunately, we can also play Bell's game with other particles than photons. Two groups of physicists in the US have used ions (atoms that have lost some of their electrons) to win Bell's game with sufficient margin to close the detection loophole.[2] It took more than twenty years to deal with this loophole, a situation that clearly illustrates the technical difficulties involved in these experiments.

Box 10. Detection Loophole. Let p be the probability that Alice's box produces a result and assume that Bob's box produces a result with the same probability. Hence, both boxes produce a result at a given time with probability p^2. In this case, Alice and Bob win Bell's game $2 + \sqrt{2} = 3.41$ times out of 4. The probability that no result is produced at a given time is $(1 - p)^2$. In this case, Alice and Bob count these as 0 and therefore win 3 times out of 4. When just one box produces a result at a given time, which has probability $2\,p\,(1 - p)$, Alice and Bob win half the time, hence 2 times out of 4. Therefore, on average, Alice and Bob's success rate is

$$p^2\left(2 + \sqrt{2}\right) + 2p\,(1 - p) \times 2 + (1 - p)^2 \times 3,$$

which is greater than 3 if and only if p is greater than

$$\frac{2}{1 + \sqrt{2}} = 82.8\ \%.$$

[2] M.A. Rowe, et al.: *Experimental violation of Bell's inequalities with efficient detection*, **149**, 791–794 (2001); D.N. Matsukevich, et al.: *Bell inequality violation with two remote atomic qubits*, Phys. Rev. Lett. **100**, 150404 (2008). Recently experiments with photons have also been realized: M. Giustina, A. Mech, S. Ramelow, B. Wittmann, J. Kofler, J. Beyer, A. Lita, B. Calkins, T. Gerrits, S.W. Nam, R. Ursin, A. Zeilinger: *Bell violation using entangled photons without the fair-sampling assumption*, Nature **497**, 227 (2013); B.G. Christensen, K.T. McCusker, J.B. Altepeter, B. Calkins, T. Gerrits, A.E. Lita, A. Miller, L.K. Shalm, Y. Zhang, S.W. Nam, N. Brunner, C.C.W. Lim, N. Gisin, P.G. Kwiat: *Detection-loophole-free test of quantum nonlocality, and applications*, Phys. Rev. Lett. **111**, 130406 (2013).

Locality Loophole

Another major difficulty in any experimental demonstration of Bell's game is the need for strict synchronisation. Alice's box must produce the result a before Bob's choice could possibly be communicated to it, either deliberately or otherwise, either overtly or in some hidden way. Now relativity theory imposes a maximal speed for any communication, namely the speed of light. So the time elapsed from the moment Bob makes his choice y up until the time when Alice's box produces the result a must not be greater than the time taken by light to cover the distance separating Bob from Alice's box. Conversely, no information about Alice's choice must have time to reach Bob's box before it produces the result b. Otherwise, this opens what is known as the locality loophole. Indeed, Alice and Bob would be 'locally connected' in the sense of relativity theory.[3]

To close the locality loophole, we must play Bell's game (and win it more often than 3 times out of 4) while ensuring that Alice and Bob are far enough apart and well synchronised. Physicists would say that they must be separated by a spacelike interval. Note that this separation concerns the whole time interval at Alice's location from the time when the choice x is made up until the time when a result a is recorded (x and a are classical variables so not subject to quantum indeterminacy). The whole of this interval must be separated in the above sense from the whole of the corresponding interval for Bob.

To illustrate the technical difficulty, imagine that Alice and Bob are about ten meters apart, as in the famous experiment by Alain Aspect described below. Light takes 30 billionths of a second to cover this distance. It is easy to see the challenge involved in making a choice, doing what is required to carry out the measurement (equivalent to pushing the joystick), and recording the result in such a short time. Obviously, there is no question of having someone make a free choice, still less of pushing a joystick, but even with modern optoelectronic equipment, a distance of 10 m is too short. One would need hundreds of meters, or better a few kilometers. Or one would need to be as smart as a physicist.

But before we consider how Alain Aspect and his group got around this difficulty, let us note that the vast majority of Bell test experiments (physicists don't speak of Bell's game, Bell test sounds more serious) pay little attention to this loophole. One reason is precisely that it is so difficult to deal with, but

[3] Those who feel worried about relativity theory may find it useful to note the following point: if a light signal cannot connect two events in one inertial reference frame, then it will not be able to in any other such frame, so this is a frame-independent concept.

a more prevalent one is that the scientists who set up these experiments know very well that, in order to prevent two students from copying one another to get through their exams, like Alice and Bob's boxes in order to win Bell's game, it is really not necessary to locate them at a spacelike separation from one another. It is enough to ensure that there is no plausible way for them to influence one another.

To get around the problem of working in a laboratory that measures only about 10 m long, Alain Aspect imagined the following strategy. Once the photons have left the source, they are directed randomly by a sort of vibrating mirror to one or other of two measurement devices. Each device always carries out the same measurement (the same choice), but since there are two devices, at the time they leave the source and move apart in space, the photons cannot anticipate which device they will be directed toward. They cannot therefore anticipate the questions they will have to answer. Through this trick, the only problem is to ensure that the orientations of the two mirrors, one on Alice's side and the other on Bob's, do indeed vibrate independently and at a high enough frequency to prevent any information on the position of one of them from influencing the result on the other side. The remaining difficulty is to ensure that the mirrors do indeed vibrate randomly and independently from one another.

Thanks to this trick, Aspect and his coworkers were able to close the locality loophole in 1982.[4] This experiment, carried out at Orsay, just south of Paris, will remain a landmark in the history of physics. Since then, several other experiments have also closed this loophole. In 1998, Anton Zeilinger, then at the university of Innsbruck in Austria, carried out a very elegant experiment involving a distance of several hundred meters.[5] This experiment used two quantum random generators to make Alice and Bob's choices and recorded the results locally in two computers. Each computer recorded the times of the events, the choices, and their results. Bell's game was thereby won on average 3.365 times out of 4.

In Geneva, we also closed this loophole over slightly more than 10 km using the fibre optic telecommunications network of our national operator Swisscom, between two villages, Bellevue to the north of Geneva and Bernex to the south.[6] To do this, we used a slightly different trick to the one implemented by

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[4] Aspect, A., Dalibard, J., Roger, G.: *Experimental test of Bell's inequalities using time-varying analyzers*, Phys. Rev. Lett. **49**, 91–94 (1982).

[5] Weihs, G. Jenneswein, T., Simon, C., Weinfurter, H., Zeilinger, A.: *Violation of Bell's inequality under strict Einstein locality conditions*, Phys. Rev. Lett. **81**, 5039 (1998).

[6] Tittel, W., Brendel, J., Zbinden, H., Gisin, N.: *Violation of Bell inequalities by photons more than 10 km apart*, art. cit.; Tittel, W., Brendel, J., Gisin, N., Zbinden, H.: *Long-distance Bell-type tests using energy--time entangled photons*, Phys. Rev. A **59**, 4150 (1999).

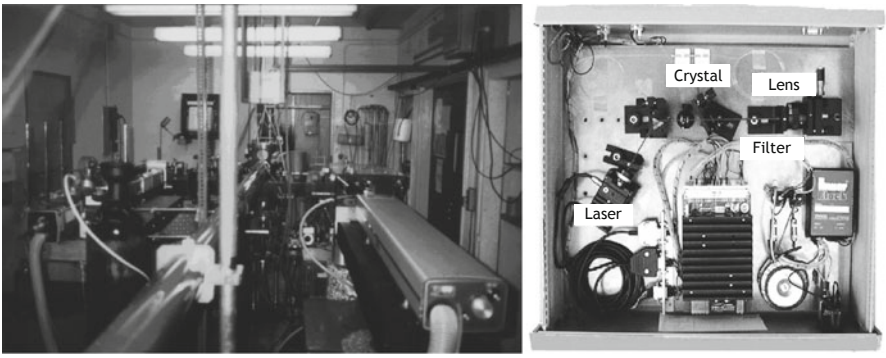

Fig. 9.1 The two parts of the figure show the enormous progress made in quantum technology. **a** Alain Aspect's laboratory in 1982, when he became the first man to win Bell's game. We see a large laboratory cluttered with bulky equipment providing the source of entangled photons used for this historic experiment. **b** The source we used in Geneva in 1997 for the first entanglement experiment to be carried out outside the laboratory, between the villages of Bernex and Bellevue. This half-empty box measures about 30 centimeters across and contains an entangled photon source that is even more efficient than the one used by Aspect. Only fifteen years separate the two experiments

Aspect.[7] At Alice's end, a semi-transparent mirror randomly directs photons to one measurement device corresponding to the left-hand joystick or to another corresponding to the right-hand joystick, and at each time, only the detectors of one of these two devices are actually active. In this way, at each time, there is only one device at Alice's end that is ready to measure the incident photon. Clearly, we lose half the photons and thus open up the detection loophole, but this was in any case already wide open due to losses in the optical fibres and the limited efficiency of the detectors. Our experiment is in fact equivalent to those in Paris and Innsbruck, but was much simpler to implement. Figure 9.1b shows our source of entangled photon pairs. Note that this little box, compatible with standard optical fibres, contains the equivalent of Aspect's whole laboratory shown in Fig. 9.1a. Advances in technology and a great deal of imagination by physicists thus led to considerable progress over the intervening 15 years!

A Combination of Loopholes

Aspect's experiment in 1982, followed by those in Innsbruck and Geneva, closed the locality loophole. Of course, in these three experiments, the detection loophole was left wide open, and in those experiments which closed the

[7] Gisin, N., Zbinden, H.: *Bell inequality and the locality loophole. Active versus passive switches*, Phys. Lett. A **264**, 103–107 (1999).

latter, the locality loophole was left open. It is thus logically possible that nature uses one or other of these two loopholes, depending on the circumstances, in order to throw us off the scent. But this is so implausible that no physicist really believes that today. Indeed, they tend to consider nature as a reliable and trustworthy partner. Nature does not cheat. As Einstein used to say, God is subtle, but he is not malicious. On the other hand, when faced with a choice between a nonlocal nature and another that obeys certain complex laws which escape us for the time being but which allow it to exploit both the detection loophole and the locality loophole at the same time, the situation is not so obvious. And since we are talking about experimental science here, the only honest response would be to carry out an experiment that simultaneously puts both loopholes to the test.

The reason why such a test has not yet been done is just that it would be very difficult. To close the detection loophole, it is better to use massive particles, these being easier to detect than photons, but for the locality loophole, photons are preferable since they propagate more easily over large distances. It thus seems that we must await an improved technology with which we can use entangled photons to distribute entanglement over large distances, then teleport this entanglement to atoms, first of all to check that the photons have in fact arrived, but also to be able to detect them efficiently. This fascinating prospect will probably become a reality within the next three years.

But the fact is that there remains a logically possible combination of loopholes and this combination must be put to the test.

A Hidden Superluminal Communication

Are there other ways out? This is a difficult question because there is always a risk of demonstrating only a lack of imagination. However, it does seem that physicists, philosophers, mathematicians, and information theorists have been trying to tackle this question now for decades and no credible alternative has been identified. But through the rest of this chapter, we shall nevertheless explore a few possibilities.

A first idea that comes to mind is of some hidden influence—hidden, that is, to the eyes of physicists at the beginning of the twenty-first century—which propagates from Alice to Bob at a speed greater than the speed of light. Somewhat surprisingly, this is how textbooks on non-relativistic physics account for Bell test experiments: the measurement made by Alice is supposed to cause a nonlocal 'collapse' of the wave function at Bob's end. This account

is not compatible with relativity but, for want of anything better, this is how we teach our students!

This assumption of a hidden influence was also John Bell's intuition when he said that everything happens as though "there is some kind of conspiracy, that something is going on behind the scenes which is not allowed to appear on the scenes."[8]

A superluminal velocity can only be defined if we make the hypothesis that this velocity is specified relative to a particular inertial frame of reference called a privileged frame. Recall that an inertial frame is a choice of spatial coordinate axes moving with constant velocity.

The assumption that there is a privileged frame goes against the spirit of relativity theory and is thus considered unholy by most physicists. However, this hypothesis of a privileged frame does not actually contradict relativity theory. To see this, we only have to note that cosmology today contains precisely such a frame, namely, the one defined since the Big Bang as the center of mass of the Universe. Physicists have even measured it with extraordinary accuracy as the frame in which the microwave background noise, a vestige of the Big Bang that still fills the whole Universe, is isotropic. Relative to this frame, the Earth is moving at about 369 km/s.[9] The direction of motion of the Earth is also well established.

The hypothesis of a privileged frame in which 'influences' can propagate faster than the speed of light should not therefore be excluded at the outset. Would this not provide some kind of explanation for nonlocal correlations? If this were the case, such correlations would not be nonlocal at all, since we should have found a local explanation, that is, a mechanism which propagates from one point to the next through space. But how could we test such a hypothesis since we do not actually know what this hypothetical privileged frame might be? To be able to carry out such a test, the basic idea is the same as for the locality loophole: Alice and Bob must make their choices and collect their results simultaneously in such a way that the hypothetical influence could not arrive in time. Either a greater distance must separate them or they must improve synchronisation. The difficulty is that one must specify the frame in which Alice and Bob have to synchronise, because according to relativity theory, if they are synchronised in one frame, there will be others in which they are not at all synchronised. There is no such problem when the speeds are less than or equal to the speed of light, because if the synchronisation is such

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[8] P.C.W. Davies, J.R. Brown (Eds): *The Ghost in the Atom*, Cambridge University Press (1986), pp. 48–50.

[9] Lineweaver, C., et al.: *The dipole observed in the COBE DMR 4 year data*, Astrophys. J. **38**, 470 (1996), `http://pdg.lbl.gov`.

that light cannot arrive in time in one frame, then it will not arrive in time in any other frame. But for higher speeds, we need to know the frame in which Alice and Bob have to synchronise.

A Swiss physicist Philippe Eberhard working at the Lawrence Berkeley National Laboratory (LBNL) near Berkeley in the United States has found a brilliant trick for testing all possible hypothetical reference frames at once. His idea is relatively straightforward. The curious reader will find a summary in Box 11. Let us simply say that the idea makes use of the Earth's 24 hour rotation and requires Alice and Bob to locate themselves along an east–west axis.

This experiment has been carried out by my group between two villages close to Geneva, Satigny to the west and Jussy to the east, these being some 18 km apart. The experiment lasted for 12 h, the time required for the Earth to make one half turn, and it was repeated four times.[10] An Italian group carried out a similar experiment.[11] The interpretation of these results is a little complicated because it depends on the speed of the Earth relative to this hypothetical privileged frame, a speed which is of course unknown. If we assume that this speed is less than the Earth's relative to the center of mass of the Universe, the experiment can then exclude any influence up to a speed of fifty thousand times the speed of light. This is already an absolutely colossal speed, far greater than most of us can conceive of, and physicists conclude that there can be no such influence. It seems therefore that there is no such "spooky action at a distance", to quote Einstein's famous comment. Once again, nonlocal correlations seem to emerge somehow from outside spacetime.

But fifty thousand times the speed of light may not be enough. Perhaps we need to repeat the experiment with greater accuracy in order to exclude speeds up to a million times the speed of light. Recall that the ratio of the speed of light to the speed of sound in air is about one million (340 m/s as compared with 300 000 km/s), so why shouldn't the next significant speed be once again a million times greater?

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[10] Salart Subils, D., Baas, A., Branciard, C., Gisin, N., Zbinden, H.: *Testing the speed of "spooky action at a distance"*, Nature **454**, 861–864 (2008).

[11] Cocciaro, B., Faetti, S., Fronzoni, L.: *A lower bound for the velocity of quantum communications in the preferred frame*, Phys. Lett. A **375**, 379–384 (2011).

Box 11. The Satigny-Jussy Experiment. Imagine that Alice and Bob are located on an east–west axis and that their measurements are simultaneous according to their clocks, i.e., they are simultaneous relative to a frame moving with Geneva. Since the Earth rotates about its axis, the Geneva frame changes all the time, but this change is very slow and hence negligible during the time required for a measurement. According to relativity theory, Alice and Bob's measurements are, in this case, also simultaneous relative to any inertial frame moving in a direction perpendicular to the Alice–Bob axis, hence relative to any frame that moves in a direction that lies in a plane passing through the North and South Poles. Over a period of 12 h, while the Earth makes half a turn about its axis, this plane will also rotate through half a turn. In doing so, it sweeps out the whole of space. So if Alice and Bob keep playing Bell's game for 12 h and if such a privileged frame does exist, there will necessarily be some time when their measurements will be perfectly simultaneous relative to this privileged frame. If Alice and Bob continue to win Bell's game more than 3 times out of 4, the explanation by a superluminal communication specified in a privileged frame will have been falsified. In practice, the synchronisation is not perfect, the alignment of the east–west axis will contain some error, and the time required to carry out the Bell experiment will not be totally negligible, so in these experiments we can only set a lower bound for the speed of this hypothetical superluminal influence.

One might also consider an influence able to travel at infinite speed, still as defined relative to some privileged frame. This is indeed possible mathematically, as was shown by David Bohm in 1952 (the year I was born).[12] However, this hypothesis implies that influences can instantaneously connect arbitrary regions of space. But how would we understand space if influences could instantaneously connect arbitrarily remote regions? In a sense, accepting such influences as an explanation for nonlocal correlations means that one accepts that these influences do not in fact propagate in our space, but follow some short cut of zero length outside our space. The explanatory power of such a hypothesis thus seems weak to me.[13] Few physicists are interested in this alternative, even though it must be said that it is viewed sympathetically by quite a few philosophers.

Some theoreticians try to get around the difficulty of experiments that will never be able to do other than establish a lower bound on the speed of these hypothetical influences by showing that, under suitable assumptions, any hidden superluminal influence must necessarily lead in the end to the possibility

[12] Bohm, D.: *A suggested interpretation of the quantum theory in terms of 'hidden' variables*, Phys. Rev. **85**, 2 (1952).

[13] To avoid having communication without transmission, the Bohm model assumes that certain variables are forever inaccessible to us. But intrinsically and forever inaccessible variables aren't physical. Interestingly Bohm himself wrote: "It is quite possible that quantum nonlocal connections might be propagated, not at infinite speeds, but at speeds very much greater than that of light. In this case, we could expect observable deviations from the predictions of current quantum theory (e.g., by means of a kind of extension of the Aspect-type experiment)", see D. Bohm and B.J. Hiley, *The Undivided Universe*, Routledge, London and NY 1993 (p. 347 of the paperback edition).

of faster-than-light communication.[14] Since that is forbidden by relativity theory, we could then conclude that there is no hidden influence at any speed whatever. This is a rich research programme that would exclude once and for all any hypotheses involving superluminal hidden influences. By a happy coincidence, while this book was being written, a group of theoreticians has managed to exclude all explanations of nonlocality that appeal to influences propagating at any finite speed (see Chap. 10).

Alice and Bob Each Measure Before the Other

I would like to discuss briefly another idea that demonstrates how physicists have tried to imagine ways out of accepting nonlocality. According to this hypothesis, put forward by Antoine Suarez and Valerio Scarani,[15] when Alice's box produces a result, it informs the rest of the Universe, and in particular, Bob's box, at a superluminal speed. And conversely from Bob to Alice. Hence the first to produce a result informs the second, which then takes it into account to win Bell's game, as in the last section. But according to this hypothesis, the superluminal speed is not defined relative to some universal privileged frame, but relative to a frame in which the emitting box is at rest (as physicists would say, in its rest frame). Indeed, each 'box', and in particular, each piece of measuring apparatus, defines an inertial frame in this way, and it is interesting to investigate the consequences of the idea that these frames would determine the speed of the information they emit.

Such a hypothesis looks hard to test. In fact, in 1997, when Suarez and Scarani put forward their hypothesis, it agreed with all the experiments that had been done up until then. But consider the following situation. Alice and Bob move apart from one another at very high speed with their respective boxes, whence the rest frame of Alice's box is not the same as the rest frame of Bob's box. Recall that, according to Einstein's theory of relativity, the chronology or time-ordering of two events viewed from two frames in motion relative to one another may be different. We can thus arrange the above experiment so that, in her frame, Alice makes here choice and collects her result before Bob, while in the same experiment, in his frame, Bob makes his choice and collects his

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[14] Scarani, V., Gisin, N.: *Superluminal hidden communication as the underlying mechanism for quantum correlations. Constraining models*, Brazilian Journal of Physics **35**, 328–332 (2005); Bancal, J.D., Pironio, S., Acin, A., Liang, Y.C., Scarani, V., Gisin, N.: *Quantum nonlocality based on finite-speed causal influences leads to superluminal signaling*, Nature Physics **8**, 867 (2012).

[15] Suarez, A., Scarani, V.: *Does entanglement depend on the timing of the impacts at the beam-splitters?*, Phys. Lett. A **232**, 9 (1997).

result before Alice. Physicists speak of a before–before experiment, since each of the two players, Alice and Bob, acts before the other! The magic of relativity theory can be used to test the magic of quantum theory.

The main difficulty in carrying out a before–before experiment is to get Alice and Bob's boxes moving fast enough to be sure that the chronological ordering of the events is indeed reversed in the two frames. This is certainly difficult, but not impossible, at least with a little imagination. It would not be very realistic to put the whole of Alice's laboratory in a spaceship. However, it should be enough to set the key component in motion, the component in which the truly chance-like event takes place. In a first experiment in Geneva,[16] we set up a detector of sorts on a disk rotating at 10 000 rev/minute, which corresponds to a tangential speed at the disk edge of 380 km/h, or about 100 m/s.[17] This may look rather slow as far as relativistic speeds are concerned, since light travels at 300 000 km/s. However, if Alice and Bob are more than 10 km apart, good synchronisation can already obtain the before–before relativistic effect. The experiment refuted Suarez and Scarani's hypothesis (although with one weak point, since the disk did not carry a true detector, only an absorber, while the information as to whether the photon was absorbed or not, equivalent to Alice's result, was read by another detector placed at the other interferometer's exit).

Antoine Suarez, who was following the experiment very closely, immediately reacted with the remark that it was not the detector that should be set in motion but the last beam splitter of the interferometer. For Suarez, it is this mirror that constitutes the choice device, that is, the component in which the choice of result is finally decided (in a truly chance-like way, as you now understand). But how could we get the beam splitter moving so fast? Well, it didn't take long for my colleague Hugo Zbinden to come up with the answer: "Let's use an acoustic wave propagating in a crystal", he suggested. Since such a wave propagates at about 2.5 km/s, the experiment can then be carried out in the laboratory. And once again, quantum theory was corroborated: even with moving mirrors, Alice and Bob win Bell's game more often than 3 times out of 4.[18] After a few difficult days, Antoine Suarez had to finally accept the result.

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[16] This experiment was financed by the *Fondation Marcel et Monique Odier de Psycho-Physique*. After doing a degree in physics and a doctorate in mathematics, Marcel Odier represents the fifth generation in his family's private bank.

[17] Stefanov, A., Zbinden, H., Gisin, N., Suarez, A.: *Quantum correlation with moving beamsplitters in relativistic configuration*, Pramana (Journal of Physics) **53**, 1–8 (1999); Gisin, N., Scarani, V., Tittel, W., Zbinden, H.: *Quantum nonlocality: From EPR-Bell tests towards experiments with moving observers*, Annalen der Physik **9**, 831–842 (2000).

[18] When Suarez learnt of our result, he immediately came to Geneva and observed that the student had set up the experiment the wrong way round: the mirrors were moving toward one another rather than moving apart! And none of us had noticed. We were not proud of ourselves! The experiment was corrected and repeated, but the result was the same.

But even though their theory was falsified (or refuted), they can be proud to have put forward a plausible scientific theory.

Superdeterminism and Free Will

So what further ways out remain to be discovered? One rather desperate hypothesis would be to deny that Alice and Bob can freely choose the direction in which they push the joysticks on their boxes. This amounts to denying the existence of free will. So if Alice does not in fact make a free choice but is programmed to push the joystick in some predetermined direction, we could imagine that Bob, or his box, already knows Alice's choice. In this case, we may also suppose that Alice's results are predetermined and that Bob, who knows everything, could easily win Bell's game. Note that he could even win with certainty, that is, more often than would be allowed by quantum physics.[19]

What a strange suggestion, to deny the existence of free will. Should we be so shocked by the idea of nonlocality as to deny one of the things we are all so intimately familiar with? We may educate ourselves, learning about mathematics, chemistry, and physics, not to mention so many other subjects, but we will never know a single equation, a single historical fact, or the slightest chemical reaction as well as what our most intimate experience is telling us. In my opinion, this is nothing but a basic error of epistemology.

If we did not have free will, we could never decide to test a scientific theory. We could live in a world where objects tend to fly up in the air but be programmed to look only when they are in the process of falling. I must admit, I have no proof that you have free will, but I certainly enjoy free will, and you will never be able to show otherwise. This kind of discussion can often go round in circles. It is logically possible, but totally uninteresting, rather like solipsism which asserts that I am the only person in the world and that everyone else is just an illusion inhabiting my own mind.

This hypothesis of superdeterminism hardly deserves mention and appears here only to illustrate the extent to which many physicists, even among specialists in quantum physics, are driven almost to despair by the true randomness and nonlocality of quantum physics. But for me, the situation is very clear: not only does free will exist, but it is a prerequisite for science, philosophy, and our very ability to think rationally in a meaningful way. Without free will, there could be no rational thought. As a consequence, it is quite simply impossible for science and philosophy to deny free will. Some physical theories are deterministic, as exemplified by Newtonian mechanics or certain interpretations

[19] Moreover, such a huge conspiracy would require extreme fine tuning in order for Alice and Bob's apparently free choices to be correlated in just the right way to 'win' at Bell's game.

of quantum theory. Raising these theories to the status of ultimate truths in a dogmatic, almost religious way amounts to a straight error of logic, since they are contradicted by our experience of free will. Note that Newton never claimed that his theory would explain everything (and that was certainly not because of any lack of ego on his part!). Quite the opposite. He stated explicitly that his theory of gravity with its nonlocal attraction at a distance was absurd, but that, for want of anything better, it could at least be used to do calculations. It was Laplace who raised Newton's theory to its quasi-religious status when he made his famous claim:[20]

> An intellect which at a certain moment would know all forces that set nature in motion, and all mutual positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

The history of quantum mechanics has been different. Its main founding father, Niels Bohr, always insisted on the completeness of his theory, even though no scientific theory can be truly complete.

In a nutshell, denying Alice the possibility of making free choices amounts to denying the relevance of science anyway. We shall thus leave this desperate hypothesis by the wayside. This should not prevent science from moving forward and providing us with a better understanding of free will, but I remain convinced that science will never completely exhaust this particular theme. And to end this section on a lighter note, let us paraphrase Newton: That free will should be an illusion, so that a man may be convinced of the existence of nonlocal correlations at a distance through empty space, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it.

Realism

To end this chapter, let us consider another desperate hypothesis, namely, the denial of realism. But what could that actually mean, and how would it help us on our way?[21]

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[20] Laplace, P.-S.: *Essai philosophique sur les probabilités*, Bachelier (1814).

[21] For some physicists, realism implies determinism. But we have seen that nonlocality implies irreducible randomness. We must therefore find some concept of realism that lives alongside true randomness.

Prior to 1990, it was almost impossible to publish a paper that referred to nonlocality or even Bell's inequality in any prestigious journal. The founding fathers of quantum physics had to fight hard to impose this new physics, and the proponents of Newtonian physics gave them a tough run for their money over many years. The next generation continued the struggle, although there were by then few opponents. In this way, they came to uphold the idea that no further progress was possible or necessary, right up until the beginning of the 1990s when applications of entanglement and nonlocality forced the physics community to make a fresh—and above all, unprejudiced—assessment of this aspect of quantum physics.[22] However, one particular habit had become well entrenched, namely, the practice of speaking and writing systematically about 'local realism' rather than 'local variables'. I suspect that this was more a matter of choosing one's words advisedly than the conclusion of any deep reflection.

It is fashionable today in some circles to say that we have the choice between nonlocality and non-realism. The first thing to do on hearing this would of course be to define what is meant by non-realism (bearing in mind that nonlocality means 'what cannot be described using only local entities').[23] Unfortunately, I am unable to tell you what non-realism is about. My feeling is that it is above all a psychological let-out: those who cannot accept nonlocality take refuge in a kind of intellectual shelter, rather like some Swiss people who are ready to disappear into their nuclear bunkers when the sirens sound. That's all very well, but one day or another, they will have to come back out.

But is there really no possible conclusion? Not quite! Go back for a moment to the Bell game. Alice and Bob's choices must be 'real', as must their results. Physicists and computer scientists would say that the inputs and outputs of Alice and Bob's boxes must be classical variables, that is, numbers (bits) that can be recognised, copied, memorised, published, in short, perfectly concrete entities that are not subject to any quantum indeterminacy. In the last section, we already discussed the hypothesis that the free choices (the inputs) might just be an illusion, but what about the results produced by the boxes (the outputs)? Could it be that these results are not real? If these results were merely an illusion occurring in our minds, we would be back with a pointless debate about some form of solipsism. Having said this, however, we may seriously wonder exactly when these results are produced. To prevent the boxes from influencing one

[22] In this context, it is interesting to note that the first publication on quantum cryptography was refused by all the physics journals! For this reason, it appeared in the proceedings of a computing conference held in India. This may seem rather surprising to an outsider, but all experienced physicists know that it is difficult to publish a particularly original idea. One must get past the barrier of skepticism within the community, a barrier that serves as an essential filter, removing ideas that are incompatible with well-established facts.

[23] Gisin, N.: *Non-realism: Deep thought or a soft option?*, Foundations of Physics **42**, 80–85 (2012).

another, these results must be produced before any possible influence has time to get there. In principle, we only have to place the two boxes far enough apart, but in practice, it is not so simple. Indeed, quantum physics is rather vague about the exact moment when a measurement result is produced. For most experimentalists, the result is already there as soon as a photon has penetrated the first few microns of the detector surface and triggered a cascade of electrons. But how can we be sure? Perhaps one should await the final amplification? Or even insist on the result being recorded in computer memory? Or human memory? Concerning this last idea, John Bell used to burst out laughing and ask whether this human memory had to belong to a physicist with a doctorate!

Although quantum physics does not tell us exactly when we can be sure that a result has been produced, it must be some time after the moment when the photon encounters a detector and well before the moment when we become conscious of this. So here then is a possible loophole, albeit very small: it could be that the results are produced much later than the experimenter imagines and that some subtle form of communication takes advantage of this to set itself up between Alice and Bob's boxes.[24]

Two physicists, Lajos Diosi and Roger Penrose, have independently developed a theoretical model that relates the duration of a measurement to gravitational effects.[25] Their models make almost the same prediction. To put it to the test, Bob must move some massive object very quickly as soon as his photon detector clicks. Recently, with my group at the University of Geneva, we have tested these models and their implication for the Bell game. The result agrees perfectly with quantum theory: neither the Diosi model nor the Penrose model provides a way out of nonlocality.[26] So it looks like quantum nonlocality is a decidedly robust concept.

The Multiverse

One last escape route that has become fashionable among certain quantum physicists assumes that there are never any measurement results. According to this hypothesis, every time we have the illusion of carrying out a measurement

[24] Franson, J.D.: *Bell's theorem and delayed determinism*, Phys. Rev. D **31**, 2529–2532 (1985).

[25] Penrose, R.: *On gravity's role in quantum state reduction*, General Relativity and Gravitation **28**, 581–600 (1996); Diosi, L.: *A universal master equation for the gravitational violation of the quantum mechanics*, Phys. Lett. A **120**, 377 (1987); Adler, S.: *Comments on proposed gravitational modifications of Schrödinger dynamics and their experimental implications*, J. Phys. A **40**, 755–763 (2007).

[26] Salart, D., Baas, A., Van Houwelingen, J.A.W., Gisin, N., Zbinden, H.: *Spacelike separation in a Bell test assuming gravitationally induced collapses*, Phys. Rev. Lett. **100**, 220404 (2008).

with N possible results, the Universe divides into N branches, each one as real as all the others, and each one hosting a different result among the N possibilities. The experimenter also divides into N copies, each one 'seeing' one of the N possible results. This is the many-worlds interpretation, or multiverse interpretation, in contrast to one that only postulates our simple universe. Supporters of this interpretation claim that their 'solution' is the simplest since it avoids the need for true randomness and they conclude that it should be accepted by applying the principle of Ockham's razor, which stipulates that one should always choose the simplest among a selection of possible hypotheses.

Everyone will reach their own conclusion about the simplicity of this interpretation. For my part, I would just make two remarks. First of all, one can always deny the existence of true randomness, whatever the theory or the experimental evidence.[27] One need only postulate that, every time this randomness manifests itself, the universe divides, and that each result actually occurs in one of the resulting parallel universes. For me, that looks very like an ad hoc hypothesis.[28] Secondly, the many-worlds interpretation implies a totalitarian form of determinism. Indeed, according to this interpretation, entanglement is never broken, but keeps on spreading further and further. So everything is entangled with everything and it leaves no room for anything like free will. The situation is even worse than with Newtonian determinism. In the latter, everything is well localised and logically separated. Newton's theory thus leaves some room for a future theory that would describe an open world, a world in which the present would not fully determine the future.[29] Indeed, this hope was in fact fulfilled with the advent of quantum theory, even though the latter falls far short of explaining free will. In contrast, the multiverse leaves no hope for an open world.[30]

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[27] Of course, one can cheat here. One can always adjoin nonlocal hidden variables to the quantum theory which determine the whole future. These parameters could simply *be* the future! They are necessarily nonlocal and hidden from our present day view. Quite frankly, that does not look very interesting to me. Once again, it's more a play on words than anything else.

[28] Followers of the multiverse idea claim that their theory is local, but it is not really clear in what sense it would be so. When Alice pushes her joystick, her box and the whole of its environment is supposed to divide into two superposed branches, each one as real as the other. Likewise for Bob. When Alice and Bob's environments meet, they entangle in just the right way to honour the rules of Bell's game in each branch. This is supposed to be an account of the dynamics described by Schrödinger's equation, but does it really do anything more than assign some somewhat vague words to a beautiful equation? Does it constitute an explanation? And above all, is it really a local explanation?

[29] For a theory including both quantum and classical variables (for example, measurement results), this can be formulated by requiring that it must be possible for the evolution of the quantum variables to be conditioned by the classical variables (the experimenter must be able to activate or otherwise a potential as a consequence of previous measurement results). L. Diósi: *Classical--quantum coexistence. A 'free will' test*, J.Phys.Conf.ser. 361, 012028 (2012); `arXiv:1202.2472`.

[30] Gisin, N.: *L'épidémie du multivers*. In: *Le plus grand des hasards. Surprises quantiques*, ed. by Dars, J.-F., Papillaut, A., Belin (2010).

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Current Research on Nonlocality

So what on earth do these two spacetime regions do to 'know' what is happening in the other region? For me, this is a very serious issue. In fact, it is even the core issue of the current conceptual revolution. So how come so few physicists seem to be bothered by it? And why was this question completely ignored between 1935, the year when the EPR paradox was published, and the beginning of the 1990s, when Artur Ekert showed that these correlations could be used in cryptography?[1] The reasons are complex. In 1935, physicists had better things to do with the new quantum physics, which suddenly provided ways to describe a host of new phenomena. Entanglement and nonlocality could wait. Then there was the influence of Bohr and his 'Copenhagen school' which stifled any incipient curiosity by proclaiming loud and clear that quantum mechanics was a complete theory.

The absurdity of such a claim was only revealed gradually, since physicists were for a long time stunned by the successes of the new physics. Indeed, how could any scientific theory ever be complete? That assumes that we are coming close to the ultimate theory, after which there will be nothing more to look for because there is nothing more to be found. What a horrifying idea! But throughout history, and especially at the ends of the last two centuries, some have believed in this possibility. The title of the book by the winner of the Nobel Prize for Physics Steven Weinberg provides a perfect illustration: *Dreams of a Final Theory*.[2] Even today, some speak seriously of a theory of everything, whose acronym TOE suggests perhaps a modicum of self-mockery. Clearly, this is not a theory we already have in our possession, but it is a significant fantasy.

Things changed at the beginning of the 1990s thanks to the efforts made by a new generation of physicists and the synergy with theoretical computer science which worked together to produce a curious and fascinating story.[3]

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[1] Ekert, A.: *Quantum cryptography based on Bell's theorem*, Phys. Rev. Lett. **67**, 661–663 (1991).

[2] Weinberg, S.: *Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature*, Vintage (1994).

[3] Rothen, F.: *Le monde quantique, si proche et si étrange*, Presses polytechniques and universitaires romandes (2012); Gilder, L.: *The age of entanglement*, Alfred A. Knopf (2008).

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How Can We 'Weigh' Nonlocality?

Now that the existence of quantum nonlocality has been firmly established, physicists are beginning to play around with it. They love to play, something that can be quite annoying to people who take themselves seriously. However, it is only by playing around with a new object, whether it is a child's toy or a scientific concept, that one can really get familiar with it. So let's play! You will certainly have noticed that this whole book turns around a game, Bell's game. And it is thanks to this game that we have been able to get right to the heart of quantum physics and its most remarkable feature, nonlocality.

Another obsession among physicists is to try to quantify, or 'weigh', everything. Naturally, nonlocality is not something that has a weight, but it is important to be able to measure it, to be able to say of two forms of nonlocality which one is the 'biggest' or the 'deepest'. For nonlocality, physicists have not yet found a good measure. It seems that, depending on which aspect of nonlocality one is investigating, there are different ways of measuring it.[4] This is clearly a sign that we have not yet fully understood this concept.

It is natural also to wonder about the idea of measuring the 'amount' of entanglement. Here a great deal of progress has been made since 1990, even though it must once again be said that many issues remain unanswered. Should we be disappointed by this state of affairs? Of course not. It is merely a sign that much remains to be discovered.

Why Not Win Bell's Game Every Time?

Quantum physics allows us to win Bell's game on average 341 times out of 400, hence far more often than 3 times out of 4, that is, far more often than if Alice and Bob's boxes produced their results locally. This has proved so fascinating to physicists that, for several generations, they have simply forgotten to ask why physics does not allow us to win Bell's game every time, that is 400 times out of 400. If nature is nonlocal, why is it not completely so? What prevents physics from predicting that we can win Bell's game every time?

It is interesting to note that this childishly simple question was first addressed only in the 1990s and that it only became a research topic in the present century. Until recently, the question was 'only': how can nature (or quantum physics, if you prefer) be nonlocal? Today, many scientific publications are exploring

[4] Méthot, A., Scarani, V.: *An anomaly of non-locality*, Quantum Information and Computation **7**, 157–170 (2007).

the consequences of a nonlocality that is much broader still than quantum nonlocality. The idea is to ask what limits quantum physics by studying it from the outside, as it were, in a much broader context than is provided by the quantum formalism.

The first theoretical toy that physicists invented for this investigation was the pair of PR boxes, named after its inventors Sandu Popescu and Daniel Rohrlich.[5] For us, these pairs of PR boxes may sound rather familiar, for they look remarkably like the boxes used by our friends Alice and Bob to play Bell's game. The difference is just that, playing with the PR boxes, Alice and Bob win every time, that is, 4 times out of 4. Nobody knows how to make such boxes so you won't be able to buy a pair (unlike the quantum boxes that allow you to win more often than 3 times out of 4).[6] However, that would not prevent physicists from playing with them. PR boxes are thus conceptual toys, or tools.

I will only give two examples to illustrate the use of PR boxes. The first is the simulation of quantum correlations. We have seen that quantum physics allows us to do far more than two measurements on a system (see Fig. 5.1 on p. 48). Two are enough for Bell's game, but physicists have the choice between infinitely many possible measurements. So must there be much more nonlocality to understand this infinite number of possibilities? Without going into the details, most of which are unknown to us anyway, let us say that with a single pair of PR boxes we may simulate all the quantum correlations corresponding to two entangled quantum bits.[7] This is rather surprising. Can we simulate all quantum correlations using PR boxes, or using other boxes capable of producing simple, or basic, correlations, while disallowing communication without transmission? The mystery remains.

A second example of the use of PR boxes comes from the field of communication complexity theory.[8] The aim is to limit the number of bits that must be communicated to carry out certain tasks. It can be shown that quantum entanglement provides no way of reducing the number of bits that need to be communicated. However, if PR boxes were available, this number could be cut down to a single bit for a wide class of problems! In short, communication complexity could be rendered trivial. This may seem rather abstract, but it would indeed be extraordinary. A single bit instead of billions of them! Unfortunately, PR boxes don't actually exist. And what if this were precisely to avoid

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[5] Popescu, S., Rohrlich, D.: *Nonlocality as an axiom*, Found. Phys. **24**, 379 (1994).

[6] `www.qutools.com`.

[7] Cerf, N.J., Gisin, N., Massar, S., Popescu, S.: *Simulating maximal quantum entanglement without communication*, Phys. Rev. Lett. **94**, 220403 (2005).

[8] Brassard, G.: *Quantum communication complexity*, Foundations of Physics **33**, 1593–1616 (2003).

such a trivialisation of communication complexity? This is indeed the view of the vast majority of information theorists for whom trivialising communication complexity is as unlikely as faster-than-light velocities for physicists. So do we thus have an explanation for the fact that quantum physics does not allow us to win Bell's game every time? This may be the case. But the problem has not yet been fully resolved. One might imagine PR boxes that are noisy enough not to trivialise communication complexity, but which would nevertheless allow us to win Bell's game more often than is authorised by quantum physics.[9]

So there we are. At some risk of losing you in the complexity of these explanations, I hope at least to have shared some of my excitement as a research physicist, the excitement of the kind of research that is currently under way. Continuing in this vein, I would like to present three more highly topical research themes. If you don't understand everything, it doesn't matter. The aim is just to understand a little more tomorrow than we did yesterday.

Nonlocality in More Than Two Parts

True randomness can manifest itself in two places. But can it also manifest itself in three, or a thousand places? The answer is not obvious, for it may be that all tripartite quantum correlations may be explicable as combinations of bipartite nonlocal randomness. Today we know that this is not the case and that there are quantum correlations which require randomness that can show up in many different places at once. But it is nevertheless true that much remains to be done in the investigation of multipartite nonlocality.[10]

A particularly interesting case occurs when several pairs of systems are entangled, e.g., A-B and C-D, and where joint measurements of the kind used in quantum teleportation (see Chap. 8) are made on systems belonging to different pairs, e.g., on B and C. It is natural to assume that the different entangled pairs are independent from one another. If there are n pairs, we speak of n-locality. This opens up a whole field of exploration which brings together the two facets of entanglement, that is, the idea of non-separable states and the idea of joint measurements.[11]

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[9] Brassard, G., Buhrman, H., et al.: *Limit on nonlocality in any world in which communication complexity is not trivial*, Phys. Rev. Lett. **96**, 250401 (2006).

[10] Svetlichny, G.: *Distinguishing three-body from two-body nonseparability by a Bell-type inequality*, Phys. Rev. D **35**, 3066 (1987); Collins, D., Gisin, N., Popescu, S., Roberts, D., Scarani, V.: *Bell-type inequalities to detect true n-body nonseparability*, Phys. Rev. Lett. **88**, 170405 (2002).

[11] Branciard, C., Gisin, N., Pironio, S.: *Characterizing the nonlocal correlations created via entanglement swapping*, Phys. Rev. Lett. **104**, 170401 (2010); Branciard, C., Rosset, D., Gisin, N., Pironio, S.: *Bilocal versus non-bilocal correlations in entanglement swapping experiments*, Phys. Rev. A **85**, 032119 (2012).

Who holds the accounts that decide who is entangled with whom? Where is the information stored about the places where nonlocal randomness can show up? Is there some kind of angel that manages the enormous mathematical space known as Hilbert space which keeps these accounts? This information would not appear to exist in our own three-dimensional space. Despite the seriousness of this childishly simple question, it has so far received little attention.

Let me tell you briefly about another highly topical field of research: what can we predict with the help of nonlocality but without using the full mathematical machinery of quantum physics? In Chap. 4, we saw that the no-cloning theorem can be given a full proof. Likewise, concerning the basis for applications like random number generators and quantum cryptography in Chap. 7. We can even recover certain features of the Heisenberg uncertainty relations.[12] On the other hand, we are still unable today to present quantum teleportation solely in terms of boxes of the kind used in Bell's game. The difficulty here lies in the joint measurements. We still cannot capture the essential features without involving the mathematical framework of quantum physics. The importance of this research has recently been recognised by Europe in the form of its DIQUIP programme[13] which brings together researchers from six countries.

The 'Free Will Theorem'

Now that any local explanation has been excluded, it is natural to ask whether there might not be some deterministic nonlocal explanation. If we cannot save locality, we might at least save determinism. Let us turn briefly to the thesis of deterministic nonlocal variables, that is, able to fully determine the results of any measurement.

In principle, this sounds feasible. Since quantum theory predicts probabilities, one might think that it would be enough consider statistical mixtures of these deterministic variables in order to reproduce the quantum probabilities. This is in fact the underlying idea of commercial programs simulating quantum phenomena as used by our students. So how does that work?

Recall that, for two widely separated events in space, the chronology may depend on the frame of reference used to describe the two events. So apart from illustrating quantum phenomena on the computer, as mentioned above, extra deterministic nonlocal variables are only useful if they make the same

[12] Scarani, V., Gisin, N., Brunner, N., Masanes, L., Pino, S., Acin, A.: *Secrecy extraction from no-signalling correlations*, Phys. Rev. A **74**, 042339 (2006).

[13] Device independent quantum information processing `www.chistera.eu/projects/diqip`.

predictions in all frames. Such variables are said to be covariant. We shall see that this is in fact impossible,[14] whereupon there cannot be covariant deterministic nonlocal variables. This therefore spells the end for determinism!

To show that such nonlocal deterministic variables do not exist, we must assume that Alice and Bob have free will. Some have thus concluded that if we human beings really do have free will, then quantum particles like electrons, photons, atoms, and so on, must also necessarily have it. This striking way of presenting the result is due to the British and American mathematicians John Conway and Simon Kochen (experts in marketing it would seem), who refer to it as the free will theorem.[15]

Let us once again argue by *reductio ad absurdum*. The demonstration is a little involved so if you get lost on the way, just go to the conclusion. Imagine therefore that Alice and Bob are playing Bell's game and look at what is going on from a frame in which Alice pushes her joystick slightly before Bob. Let k be the nonlocal variable which, by hypothesis, determines the results produced by Alice and Bob's boxes. Hence Alice's result a depends on this variable k and on her choice x. We write $a = F_{AB}(k, x)$, where F_{AB} is a function. Viewed from this frame, when Bob pushes his joystick, his result b may depend on the variable k and his choice y, but also on the choice x made by Alice. We write $b = S_{AB}(k, x, y)$. This is where we see that the variable k is nonlocal,[16] since Bob's result can depend on Alice's choice. Note that the symbols F_{AB} and S_{AB} indicate 'first' and 'second' in the chronological order AB.

Now consider the same situation when viewed from another frame in which Bob pushes his joystick slightly before Alice. For example, this second frame may be carried by a rocket moving very fast from Alice toward Bob. In this case, Bob's result b depends only the variable k and his choice y, so we write $b = F_{BA}(k, y)$. However, Alice's result a can now depend on the nonlocal variable k, her choice x, and Bob's choice y, so we have $a = S_{BA}(k, x, y)$. Once again, the symbols F_{BA} and S_{BA} indicate 'first' and 'second' in the chronological order BA.

But Alice's result a cannot depend on the frame used to describe the experiment (the game). Therefore, we must always have $a = F_{AB}(k, x) = S_{BA}(k, x, y)$. The latter equality can only be satisfied if S_{BA} does not in fact depend on y, whence Alice's result does not in fact depend on Bob's choice. Likewise, Bob's result cannot depend on Alice's choice. But this was the locality condition as

[14] Gisin, N.: *Impossibility of covariant deterministic nonlocal hidden variable extensions of quantum theory*, Phys. Rev. A **83**, 020102 (2011).

[15] Conway, J.H., Kochen, S.: *The free will theorem*, Found. Phys. **36**, 1441–1473 (2006).

[16] More precisely, a variable is not intrinsically local or nonlocal. In the present case, physicists say that the variable k is nonlocal due to the use made of it by the function S_{AB}.

formulated by Bell in 1964: Alice's box produces its result locally, and likewise for Bob's box. In this case, as we have seen, Alice and Bob cannot win Bell's game more often than 3 times out of 4. And that means that, if they do in fact win more often than 3 times out of 4, this also excludes the possibility of nonlocal variables that are both deterministic and covariant.

To sum up then, the only remaining possibility is non-deterministic nonlocal variables. This is how quantum theory describes Bell's game. Note that here 'non-deterministic' is yet another negative qualifier. It does not tell us what these variables are, nor how these variables or these models describe Bell's game. It merely asserts that it will not be deterministic. In particular, non-deterministic does not mean probabilistic in the usual sense of probabilities, for it is not a statistical mixture of deterministic cases. (A good illustration of this research can be found in papers by Colbeck et al. and Pusey et al.[17])

A Hidden Influence?

I cannot resist describing one last result, also very recent, even if it is once again negative. To save locality, that is, to save the idea that things and influences propagate continuously from one point to the next, without leaps or breaks, an idea so deeply entrenched in us that it is very difficult to go beyond it, it is very tempting to imagine that Alice or her box influences Bob in some subtle way that remains hidden to the eyes of physicists at the beginning of the twenty-first century. Or indeed, that Bob influences Alice, depending which one of them makes first choice. Since this chronology depends on an arbitrary choice of frame, it is tempting to imagine that there is some privileged frame which determines once and for all the time ordering of all the relevant events. We have seen that experiment can establish a lower bound for the speed of such an influence (see Chap. 9). But could it not be that the apparent nonlocality might be due to an influence propagating continuously from one point to the next between Alice and Bob at some gigantic speed defined in a privileged frame that today's physics has not yet identified? According to this hypothesis, if the influence arrives in time, the observed correlations are those predicted by the quantum theory, but if the influence does not arrive in time, then the correlations are necessarily local, so they would not allow us to win Bell's game. Such a hypothesis does not respect the spirit of Einstein's relativity theory, but

[17] Colbeck, R., Renner, R.: *No extension of quantum theory can have improved predictive power*, Nature Communications **2**, 411 (2011); Pusey, M.F., Barrett, J., Rudolph, T.: *The quantum state cannot be interpreted statistically*, Nature Physics **8**, 476–479 (2012).

it would not contradict any experimental tests of that theory. In short, this hypothesis maintains a peaceful coexistence with relativity theory, just like the nonlocal quantum correlations that allow us to win Bell's game.

At the outset, it looks impossible to exclude such an explanation. At best we can carry out experiments like the one described in Chap. 9 which establish a lower bound on the speed of this hypothetical influence. But we can be cleverer than that.

Would the existence of influences propagating faster than light necessarily imply that we could communicate faster than light? One might imagine that such influences could remain forever hidden. That does not sound very physical, but it is natural to think that, as long as physicists are unable to control these hypothetical influences, they cannot use them to communicate faster than light.

But surprisingly, this simple hypothesis according to which one cannot communicate faster than light without controlling these influences is enough to prove that such influences cannot exist! This result was obtained during the writing of this book by my student Jean-Daniel Bancal, a Malaysian postdoctoral student Yeong-Cherng Liang, and three of my former colleagues, Stefano Pironio, now in Brussels, Antonio Acin in Barcelona, and Valerio Scarani in Singapore, together with myself. This was the culmination of a great adventure that began more than 10 years ago. Since it took us all that time, don't be surprised to find it a little complicated. I will try to summarise these developments, but you could also jump directly to the conclusion. Understand simply that even the assumption of influences propagating an arbitrary finite speed, faster than light but nevertheless finite, even this extreme hypothesis can be excluded. Nature is definitely nonlocal.

The hypothesis of superluminal influences can reproduce all the experimental results between two partners like our friends Alice and Bob. Indeed, since no experimental synchronisation is perfect, we can always assume that these influences travel fast enough to correlate two events. With three partners, the question remains open.[18] But with four partners, call them A, B, C, and D, we have found the following argument. Imagine that, in the privileged frame, A carries out her measurement first, then *D*, then almost at the same time B and C, in such a way that the influence from A arrives in time at the three other partners and the influence from D arrives in time at B and C, but the latter two cannot influence one another. In this particular situation, the correlations ABD and ACD are those predicted by quantum theory, according

[18] After finishing this book, this question was answered in T.J. Barnea, J.D. Bancal, Y.C. Liang, and N. Gisin: *Tripartite quantum state violating the hidden-influence constraint*, Phys. Rev. A **88**, 022123 (2013).

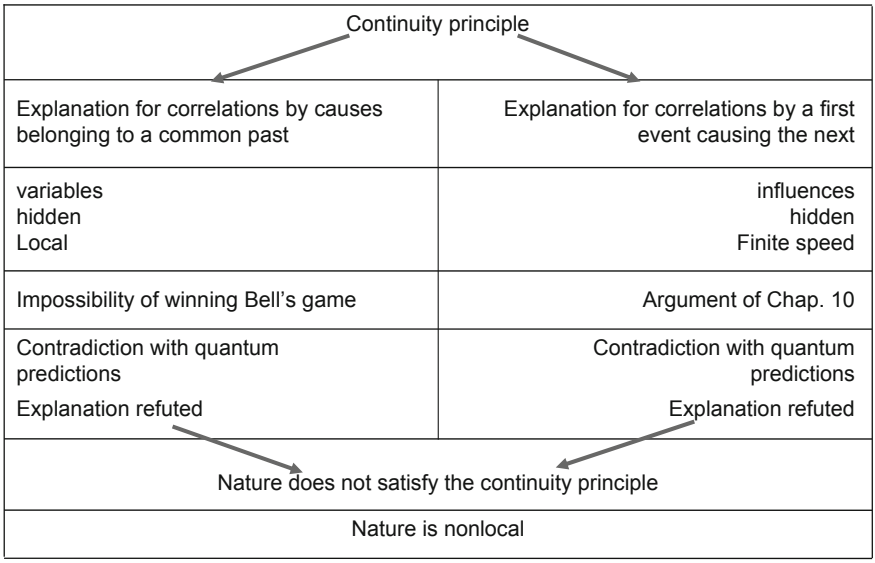

Fig. 10.1 John Bell's research programme. Today the programme has been taken to completion. There is no possible local explanation for certain correlations that can be produced by quantum physics. Nature is nonlocal and God does play dice in order to allow a form of nonlocality that excludes communication without transmission

to the hidden influence assumption. However, the correlation BC is local. But we have found a very surprising inequality[19] satisfied by all the four-part correlations where BC is local and which cannot be used for communication without transmission. Furthermore, this inequality only contains terms involving correlations between ABD and between ACD. Each of these triplets of parts is interconnected by the hidden influence hypothesis, so that A influences D which influences B, for example. Hence, in the above configuration, any model using finite speed hidden influences predicts the same value as quantum theory for this inequality. But the quantum prediction violates our inequality, so we may conclude that any model appealing to finite speed hidden influences necessarily produces correlations allowing superluminal communication.

The result sketched above completes the programme initiated by John Bell, to explain quantum correlations through a principle of continuity according to which everything propagates continuously from one point to the next in space. Figure 10.1 illustrates this programme. Once again, the inescapable conclusion is that remote events are connected in a discontinuous way, whence nature is indeed nonlocal.

[19] Bancal, J.D., Pironio, S., Acin, A., Liang, Y.C., Scarani, V., Gisin, N.: *Quantum nonlocality based on finite-speed causal influences leads to superluminal signaling*, Nature Physics **8**, 867 (2012), arXiv:1210.7308.

Conclusion

So we come to the end of this book. As I warned you, you won't have understood everything. Nobody knows why quantum physics is nonlocal. On the other hand, you will have understood that nature is not deterministic and that it is capable of genuine acts of pure creation. Put another way, it can produce truly chance-like events. Moreover, as soon as we have taken on board the idea that these events are irreducibly chance-like and not just something pre-existing that was hidden from us, we understand that nothing can prevent this randomness from cropping up in several places at once without that implying any communication between those places.

These places are not arbitrary but must first be entangled. Entanglement is carried by quantum objects such as photons or electrons, and these objects propagate at finite speeds, less than or equal to the speed of light. In this sense, the notions of distances and space remain relevant, even though nonlocal randomness can show up in two arbitrarily widely separated places.

I have said in this book that nonlocal correlations seem to emerge somehow from outside of space and time in the sense that no story taking place in space as time goes by can account for the way nature produces such correlations. Effectively, it is true that there is no ordinary story, telling us how things and events can influence each other, move around, and propagate continuously from one point to the next, that would be able to describe the appearance of nonlocal correlations. But does that mean that physicists must abandon all their endeavours to understand nature? It always surprises me that many physicists do not appear much concerned about this question. They seem satisfied by being able to do the necessary calculations. Perhaps these physicists would say that computers understand nature?

And yet science has always been characterised by the quest for good explanations.

Until the advent of quantum physics, all the correlations predicted and observed in science have been explained by causal chains propagating contiguously from one point to the next, that is, by local arguments. All these prequantum explanations are also characterised by determinism. In principle, everything is determined by initial conditions. Although it is often impossible

N. Gisin, *Quantum Chance,*
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in practice to following the details of these deterministic causal chains, physicists never doubted their existence. But quantum physics forces us to formulate a new kind of good explanation for nonlocal correlations.

But how can we explain or account for nonlocality? With our prequantum conceptual tools, it would be impossible, so we are compelled to extend our toolbox. One way is to speak about the nonlocal randomness produced by entangled objects.

Imagine some kind of conceptual dice, rather like the PR boxes discussed in Chap. 10, which can be 'thrown' by both Alice and Bob and whose random result occurs both with Alice and with Bob. This nonlocal dice is thrown by Alice's choice or by Bob's choice to carry out a measurement, hence by pushing the joystick on Alice's box or on Bob's. Slightly more formally, the random process can be initiated either by Alice by choosing her input x or by Bob choosing his input y. The results a and b are random, but with the guarantee that they somehow 'relate to' (or 'attract') one another in such a way as to favour the underlying correlation of the Bell game as expressed in the relation $a + b = x \times y$. If we allow this kind of explanation, we can then understand nonlocality, just as we eventually understood universal attraction by accepting to say that all mass, and in particular all humans, are attracted by the Earth. Naturally, when it comes to universal attraction, we have the familiar parallel with the fridge magnet. If quantum cryptography ever becomes as familiar, we will be able to say to our children: "You see, nonlocality is just like in quantum cryptography. It isn't Alice that sends a secret key to Bob, nor Bob that sends a secret key to Alice, it's Alice and Bob who, while far apart, produce a secret key together which materialises simultaneously with each of them."

Is that the only way to account for nonlocal randomness? Some prefer to speak of backward causation, in the sense that Alice's choice acts back in time on the source of entanglement, which in its turns acts forward in time on Bob's quantum system. So backward causation acts backwards in time, toward the past. It propagates from one point to the next through space, but toward the past. Personally, I have no doubt that nonlocality does, like relativity theory, generate some difficulties for our familiar concept of time, but from there to imagining a chronologically reversed causality which goes back in time seems a drastic step!

I mention this approach as an illustration of today's research. You will have understood that I prefer my own explanation based on the idea of nonlocal randomness that can manifest itself in several places at once, regardless of the distance between them, but it may be that the future will surprise me and that future generations will employ a quite different form of explanation. But one thing is certain: we will account for nonlocality. Physicists will never abandon

their great enterprise to understand the world, and they will come up with an explanation for this too.

Nonlocal randomness is thus a new explanatory mode to be included in our conceptual toolbox along with all the other devices we have accumulated over the centuries for understanding our world. And this is a genuine conceptual revolution! Since quantum theory predicts the existence of nonlocal correlations, we have no choice but to make do with them and adopt this new kind of explanation.

Quantum nonlocality took a long time to get accepted as a central concept of physics. Even today, many physicists reject the term 'nonlocal'.[1] And yet, from as early as 1935, Einstein and Schrödinger, among others, maintained that this aspect of quantum theory is its main feature. It would seem that what all these suspicious physicists have not grasped is that quantum nonlocality does not allow communication. Nothing goes from Alice to Bob, or indeed from Bob to Alice. It's just that a random event manifests itself in several places in a way that cannot be described locally, that is, in a nonlocal way. Einstein was wrong to speak of 'nonlocal action', since there is no action of Alice on Bob or of Bob on Alice. But he was quite right to stress the significance of this aspect of quantum theory, for it is indeed the one that best distinguishes quantum physics from classical physics. Today, when we would like to be sure that a system is a quantum system, we must show that it can be used to produce nonlocal correlations, in other words, that it could be used to win Bell's game. Today, violation of a Bell inequality is the very signature of the quantum world.

But this is still a serious blow to our intuitions. Will the quantum technologies now under development make quantum physics and its nonlocality intuitive to us one day? I would say they will. We could usefully begin by dropping the old-fashioned term 'quantum mechanics' and replacing it systematically by 'quantum physics'. There is nothing mechanical about this particular branch of physics!

Let us sum up the main points once again. We have seen that the nonlocal correlations and the existence of true randomness are very closely related. Without true randomness, nonlocal correlations could necessarily be used to communicate without transmission (hence at arbitrary speeds). So the central

[1] Things have changed considerably over the past twenty years. The emergence of quantum information and conversion of the enormous solid state physics community have seen a proliferation in the use of words that were almost forbidden two decades ago, such as 'nonlocality', 'nonlocal correlations', 'true randomness', and 'Bell's inequality'. But there remains a large community, the high energy physics community, that refuses to budge. These physicists seem to think that only their own branch of physics treats fundamental issues and that the rest of physics is merely glorified engineering. The numbers of professional physicists increased significantly in the twentieth century, and the sociology of this community remains to be written.

concept of this book necessarily implies the existence of true randomness and hence the end of determinism. Conversely, once the existence of true randomness has been accepted, the existence of nonlocal correlations no longer looks as crazy as classical physics with its stubborn determinism would have us believe. Indeed, if nature is really capable of producing truly chance-like events, why should the correlations observed in nature be limited to local correlations?

The impact of nonlocality on metaphysics, that is, on the world view suggested by modern physics, cannot be overestimated. It took centuries for the atomistic view to predominate in Europe. This was a world made up of vast numbers of atoms, like tiny invisible beads which assemble in different ways to make up all the objects known to us. Their erratic motions produce the feeling of heat and supply energy in the steam engines which gave us the industrial revolution. At the time, in China, it seems that this world view would not have found much support in intellectual circles. They felt that, in a world full of emptiness between the atoms, we could neither see nor hear, for the perception of our senses would be inhibited by the empty spaces.[2] Apparently, in ancient Chinese metaphysics, action at a distance was perfectly natural and belonged to a universal harmony that related everything to everything else. Quantum physics does not support such a holistic world view. In quantum physics, everything is not entangled with everything else and only a few rare events are correlated in a nonlocal way. And above all, at risk of repeating myself, there is no cause here that acts over there. Entanglement is a kind of 'probabilistic cause' whose effects can show up in several places, without allowing communication at a distance. Entanglement determines the natural tendency of objects to produce such and such correlated responses to certain questions. These responses are not predetermined, they are not inscribed in the state of the object. It is just the propensity to produce such and such a result that is inscribed in the state of the object.

Personally, I don't find it so extraordinary that a quantum object should not contain within itself all the answers to all the questions that physicists might ask of it, but that it should possess only a propensity to produce those answers. I don't find it difficult to accept that the world is not deterministic. This world, full of propensities and chance-like events obeying well defined laws, is to my eyes much more interesting than a world where everything is perfectly predetermined, and has been so since the beginning of time.

But you can bet that there remains a great deal more to learn about the world. In particular, we still don't understand how to make this compatible with Einstein's theory of relativity. And nor do we understand the whole

[2] Needham, J.: *Science in Traditional China*, Harvard University Press, 1981.

mathematical structure, nor the full potential of applications in information processing, nor—and this is perhaps the most surprising thing—the limits to nonlocality: why does quantum physics not allow more nonlocality?

This last question is a good illustration of the distance we have come since Einstein, Schrödinger, and Bell. In those days, the question was: Do the nonlocal correlations predicted by quantum theory really exist? Today, no physicist could doubt this. The problem now is to integrate this into a relativistic theory and to understand the limits on nonlocality. What we need to do is to study quantum nonlocality from outside quantum theory. And we're working on it.